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Modified Hybrid Map (MHM) To Enhance Cryptosystem: A Chaotic Dynamical Systems Approach

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Abstract **Original Research Articles**

Chaotic maps such as the quadratic and tent maps have demonstrated great potential in constructing secure and lightweight cryptosystems due to their sensitive dependence on initial conditions and ergodic behavior. However, quadratic maps suffer from limited parameter ranges and quickly settle into periodic behavior, reducing unpredictability and weakening their cryptographic strength over time while tent maps display symmetry and linearity that can be exploited analytically, making them vulnerable to attacks if not properly hybridized or perturbed. In this paper, we present a detailed investigation of these maps as core elements of a secure chaotic cryptosystem. We analyze their dynamical behavior using bifurcation diagrams, Lyapunov exponent plots, time series analysis and entropies in order to study the sequential behaviors of the maps. Furthermore, we hybridized the two maps to form a new chaotic map named Modified Hybrid Map (MHM). Our results show that both maps offer unique advantages in security design, and the integration of the two maps improves randomness, confusion, and diffusion properties which are very essential in cryptography.

Keywords: Chaotic Maps, Tent Map, Quadratic Map, Cryptography, Bifurcation Diagram, Lyapunov Exponent, Dynamical Systems, Time Series, Entropy.

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1. INTRODUCTION

Cryptography seeks to secure communication through the encoding of information in a way that prevents unauthorized access. Traditional cryptographic methods often rely on computational complexity, but chaos-based cryptography leverages the inherent unpredictability and sensitivity of chaotic systems.

Among these chaotic maps are: the quadratic map and the tent map which are widely studied due to their mathematical simplicity and rich dynamical behavior. Quadratic and tent maps, while simple and capable of generating chaotic behavior, suffer from limited key space and predictable patterns under certain parameters, making them vulnerable to cryptanalysis. Additionally, their low-dimensional structure and lack of complexity can hinder robustness and security in advanced cryptographic applications. To enhance performance, especially in cryptographic applications, hybridizations of chaotic maps have been proposed.

This paper tends to hybridize the two simplest yet dynamically rich chaotic systems: the quadratic map and the tent map. These maps exhibit bifurcations, positive Lyapunov exponents, entropies and time series analytical behavior across specific parameter ranges, making them ideal candidates for secure encryption applications.

2. MOTIVATION

The hybridization of quadratic and tent maps is motivated by the inherent limitations observed in each map when used individually for applications such as cryptography and complex system modeling. The quadratic map, although capable of producing chaotic behavior, is sensitive to parameter changes and often exhibits periods of stability within specific intervals, reducing its unpredictability. Similarly, the tent map, while piecewise linear and computationally efficient, suffers from limited complexity and can produce symmetrical patterns that weaken its security robustness.

To overcome these drawbacks, hybridizing both maps combines their complementary strengths, leveraging the strong nonlinearity and bifurcation properties of the quadratic map with the uniform distribution and sharper transitions of the tent map. This integration enhances the entropy, complexity, and overall chaotic behavior of the resulting system, making it more suitable for secure communication, pseudorandom number generation, and data encryption. Thus, the hybrid map offers improved dynamical diversity and greater resistance against statistical and differential attacks, which are critical in cryptosystem design.

From the foregoing, it is obvious that both quadratic and tent maps have peculiar deficiencies that make them not suitable for systems where high security is required. Hence, the need for a hybridized map that can bridge the notable gaps.

3. RELATED WORKS

3.1 Mathematical Background

Chaos theory studies the behavior of dynamical systems that are highly sensitive to initial conditions, a phenomenon popularly referred to as the "butterfly effect." In discrete-time systems, maps such as the tent and quadratic maps provide straightforward frameworks for generating chaotic sequences.

3.2 Discrete Dynamical Systems

A discrete map (or discrete dynamical system) is a mathematical function that describes how a point in a space evolves over discrete time steps.

A discrete map defines the evolution of a system as represented in Equation 1:

$$x_{n+1} = f(x_n) \quad (1)$$

Where

x_n is the state of the system at time step n and f is the map (or rule) that determines how the state changes from one time step to the next.

3.3 The Quadratic Map

The quadratic map is a well-known one-dimensional iterative function that exhibits period-doubling bifurcations and routes to chaos for varying parameter (Strogatz, 2018). Though structurally simple, it demonstrates complex dynamical phenomena such as bifurcation cascades and sensitive dependence on initial conditions. It is a nonlinear discrete dynamical system defined by Equation 2 as:

$$x_{n+1} = r(x_n^2 - 1) \quad (2)$$

Where r is a control parameter. It exhibits a wide range of behaviors, from stable fixed points to periodic and chaotic dynamics, depending on the value of r. The map is sensitive to initial conditions and is commonly used in chaos theory and cryptography for its unpredictability.

However, quadratic map has some notable limitations. Firstly, its chaotic regime is limited to specific values of the parameter r, beyond which the system can diverge to infinity, making it less stable for broad applications. Secondly, its unimodal and smooth structure can make it predictable under certain numerical or analytic attacks in cryptographic systems.

Additionally, due to its symmetric and deterministic nature, the quadratic map may lack sufficient complexity when used alone in secure communications, requiring hybridization with other maps for enhanced security.

Its lack of uniform invariant density and limited chaotic range reduce its utility in secure encryption systems (Li et al., 2005).

3.3 The Tent Map

The tent map is a piecewise linear discrete map defined in Equation 3 as:

$$x_{n+1} = \begin{cases} rx_n, & if\ x_n < 0.5 \\ r(1 - x_n), & if\ x_n \geq 0.5 \end{cases} \quad (3)$$

It is shaped like a triangle ("tent") and maps the interval [0,1] to itself with a well-known uniform invariant density making it desirable for applications requiring pseudo-randomness and ergodicity (Alligood et al., 1996). The tent map is also computationally efficient.

The behavior of the system depends on the control parameter r, typically in [0,2].

For certain values of r, the tent map exhibits chaotic behavior with high sensitivity to initial conditions.

It is commonly used in modeling, encryption, and studying deterministic chaos. Nevertheless, its linear structure and finite state space may limit unpredictability in cryptographic use when compared to more nonlinear chaotic maps. It is easy to analyze and potentially predictable, which can reduce its effectiveness in cryptographic applications. Additionally, the tent map's maximum entropy and chaotic behavior are highly dependent on precise parameter tuning; small deviations can lead to loss of chaos or degeneracy. Furthermore, it lacks the deep nonlinear complexity found in other maps, limiting its standalone capacity to generate strong pseudo-random sequences for high-security systems.

Hybridization aims to merge the nonlinearity of the quadratic map with the uniform density and ergodic properties of the tent map. Recent works ((X. Wang et al., 2017); (Jangir & Yadav, 2021)) show that hybrid chaotic maps significantly increase key space, entropy, and unpredictability which are key metrics in securing communication systems.

Hybrid maps have been demonstrated to have broader chaotic regions, enhanced Lyapunov exponents, and improved entropy measures. For instance, (Jangir & Yadav, 2021) showed that the hybrid map outperforms individual maps in resisting statistical and differential attacks in chaos-based encryption schemes.

3.4 Observation From Review of Related Works

1. Hybrid maps require careful parameter tuning; otherwise, they may lose chaotic behavior and exhibit periodic or unstable dynamics (Wang et al., 2021).
2. The increased complexity from combining two or more maps leads to higher computational demands and longer processing time (Zhang & Chen, 2020).
3. Some hybrid maps can experience transitions from chaos to regular behavior under certain parameter settings, making long-term prediction difficult (Li et al., 2022)

4. Due to the novelty of many hybrid models, they often lack standardized analytical frameworks for evaluating bifurcation, Lyapunov exponents, or entropy metrics (Kumar & Singh, 2019).

The hybridized map is characterized as follows:

- Nonlinear and piecewise structure for richer dynamics
- Higher complexity and unpredictability
- Increased sensitivity to initial conditions
- Broader chaotic range over parameter values
- Better randomness and entropy generation

3.7 Applications and Evaluations

Quadratic and tent maps have been integrated into image encryption algorithms, random number generators, and watermarking systems (Pareek et al., 2006). However, individually, they face challenges such as periodic windows and deterministic predictability. Hybrid maps, by comparison, generate more complex dynamics, eliminating weaknesses inherent in standalone maps.

Studies have evaluated these maps using metrics such as bifurcation diagrams, Lyapunov exponents, entropy analysis, and statistical correlation coefficients. Hybrid maps consistently show superior performance in these evaluations, underscoring their potential for advanced cryptographic protocols (Khadir & Oubrahim, 2020).

4. METHODOLOGY

This section details the systematic approach adopted to design and implement a **hybridized Quadratic-Tent Map (QTM)** for chaotic signal generation, a core component of the proposed cryptographic framework. The methodology is divided into three stages: (i) defining the hybrid map, (ii) implementing the chaotic sequence generator, and (iii) validating the system through statistical metrics such as Lyapunov Exponent, entropy, and bifurcation analysis.

Hybridized Chaotic Map

A hybridized chaotic map combines two chaotic maps *(the quadratic map and the tent map)* to form a single system with enhanced dynamical behavior. It inherits features from both maps, like the nonlinearity of the quadratic map and the sharp switching behavior of the tent map.

The hybrid system is defined by Equation 4 as follows:

Let quadratic map $Q(x) = r(x^2 - 0.5)$ and

Tent map $T(x) = \begin{cases} rx, & if\ x < 0.5 \\ r(1-x), & if x \geq 0.5 \end{cases}$ then

$$H(x) = T(Q(x)) = \begin{cases} r(r(x^2 - 0.5)), & if\ r(x^2 - 0.5) < 0.5 \\ r(1 - r(x^2 - 0.5)), & if r(x^2 - 0.5) \geq 0.5 \end{cases}$$

$$H(x) = T(Q(x)) = \begin{cases} r^2(x^2 - 0.5)), & if\ r(x^2 - 0.5) < 0.5 \\ r - r^2(x^2 - 0.5), & if\ r(x^2 - 0.5) \geq 0.5 \end{cases}$$

In order to keep H(x) in the range of 0 and 1 we introduce modulus 1 into function H(x), hence

$$H(x) = T(Q(x)) mod\ 1 = \begin{cases} r^2(x^2 - 0.5)) mod\ 1, & if\ r(x^2 - 0.5) < 0.5 \\ r - r^2(x^2 - 0.5)\ mod\ 1, & if\ r(x^2 - 0.5) \geq 0.5 \end{cases} \quad (4)$$

Where H(x) is the hybridized map. The pseudocode represented by Algorithm 1 gives the procedure for generating sequence of random value using hybrid map H(x).

Algorithm for QTM Sequence Generation

Algorithm 1: *Hybrid Quadratic-Tent Map Generator*

Input:

- Initial condition: $x_0 \in [0,1]$
- Control parameter r
- Number of iterations, N

Output:

- Chaotic sequence $X = \{x_1, x_2, x_3, \dots, x_n\}, x_i \in [0,1], i = 1,2, \dots . n$

```
START
INITIALIZE sequence array X ← [ ]
SET X[1] ← x_0
FOR i ← 2 to N do
    IF r(X[i − 1]^2 − 0.5) < 0.5  then
        X[i] = r^2(X[i − 1]^2 − 0.5))mod 1
    ELSE
        X[i] = (r − r^2(X[i − 1]^2 − 0.5))mod 1
    ENDIF
ENDFOR
OUTPUT X
STOP
```

Implementation Environment

The implementation was carried out using **Python 3.11** with NumPy for numerical computation and Matplotlib for visualization. The system was tested over a range of initial conditions and parameter values to assess its chaotic behavior through entropy, Lyapunov exponent, time series, and bifurcation analysis.

Chaotic Behavior Validation

The following metrics were used to validate the effectiveness of the modified hybrid QTM:

- **Lyapunov Exponent** to quantify sensitivity to initial conditions.

- **Permutation Entropy** and **Approximate Entropy** for statistical complexity.
- **Bifurcation Diagram** to study transitions from periodic to chaotic behavior.
- **Time Series Analysis** to proof the enhanced chaotic behaviour.

These validation steps ensure the proposed hybrid system generates strong randomness properties ideal for cryptographic use.

5. ANALYSIS, DISCUSSION AND RESULTS

5.1 Bifurcation Diagram Analysis

Bifurcation diagrams illustrate the long-term behavior of the system as parameters vary. In chaotic cryptosystems, bifurcation helps identify ranges with high entropy and sensitivity. Figure 5.1 shows the bifurcation diagrams of the three (3) maps, Quadratic, Tent and Hybrid maps. It is obviously seen that the hybrid map combines the nonlinearity of the quadratic map with the sharp switching behavior of the tent map. The bifurcation diagram exhibits higher density and complexity, with more irregular and fragmented patterns than the individual maps. The chaotic region is extended, and the transitions between periodic and chaotic regimes appear more sudden and irregular. It shows increased entropy and unpredictability, which is beneficial for cryptographic applications. This fusion makes the system more resistant to attacks and more robust as a chaotic key generator in secure systems.

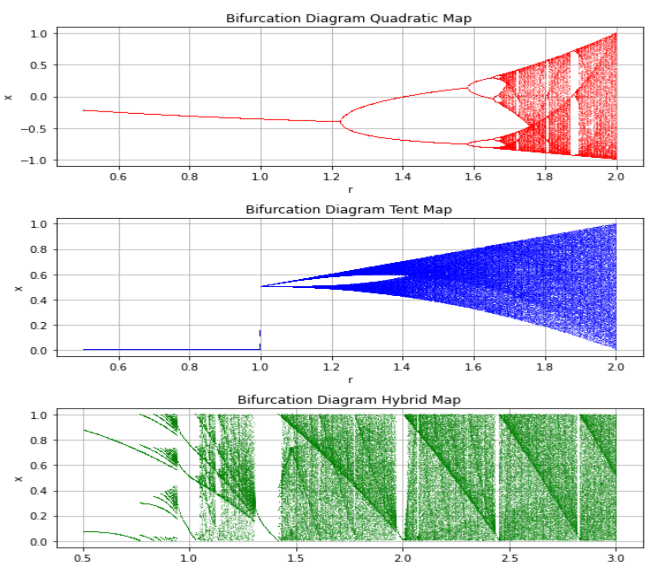

Figure 5.1: Bifurcation Diagrams of Quadratic, Tent and Hybrid (MHM) maps

Quadratic Map Bifurcation Diagram Analysis

The bifurcation diagram of the quadratic map exhibits a classical route to chaos. Initially, for small r, the system converges to a stable fixed point. As r increases, a period-doubling cascade begins, indicating bifurcations to period-2, period-4, and so on until chaotic behavior emerges beyond a critical threshold (around $r \approx 1.2$)

In the chaotic regime, the diagram shows a dense spread of points, reflecting the sensitivity of the system to initial conditions and parameter values. However, windows of periodicity are also observed where the system momentarily returns to periodic orbits amidst chaos.

This implies that the quadratic map demonstrates deterministic chaos, but its structural symmetry and potential limited range of chaos may reduce its cryptographic strength unless combined with another map.

Tent Map Bifurcation Diagram Analysis

The bifurcation diagram for the tent map reveals a strikingly different structure. As r approaches 2, the diagram quickly enters a fully chaotic state.

Unlike the quadratic map, the tent map's bifurcation diagram lacks deep period-doubling transitions. Instead, it displays a more uniform transition to chaos. For values of $r \in (1.4,2)$, the system becomes ergodic and mixing, with points evenly distributed across the space.

This implies that the tent map’s uniform chaotic behavior and wider range of chaotic r values make it suitable for cryptographic schemes requiring high statistical randomness and minimal predictability.

Modified Hybrid Map (MHM) Bifurcation Diagram Analysis

The hybrid map combines the dynamics of the quadratic and tent maps:

Its bifurcation diagram showcases a complex, rich structure, synthesizing the nonlinear bifurcation features of the quadratic map with the uniform chaos of the tent map. As r increases, the hybrid system undergoes early bifurcations and enters chaos more rapidly.

Unlike the individual maps, the hybrid bifurcation diagram displays noisy, intricate bands of chaos and irregular bursts of periodicity. This complexity translates to greater entropy and a larger key space when applied in secure systems.

It implies that hybrid map provides enhanced security by avoiding predictability and increasing randomness. Its bifurcation behavior supports robust key generation in chaos-based cryptographic applications.

5.2 Lyapunov Exponent

The Lyapunov exponent quantifies how sensitive a system is to initial conditions.

This measures the average rate of separation of infinitesimally close trajectories. Positive values indicate chaos. The Lyapunov Exponent lambda λ is a quantitative measure of the sensitivity of a dynamical system to initial conditions. It describes the average rate of divergence of nearby trajectories in phase space. A positive Lyapunov exponent indicates chaos. Small differences in initial states grow exponentially over time, whereas a negative exponent suggests convergence to stable

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points or periodic orbits.

Figure 5.2 represents the Lyapunov Exponent of Quadratic, Tent and Hybrid maps. The hybrid map's Lyapunov diagram shows a richer and more complex structure. λ becomes positive in a broader parameter range, indicating extended chaos compared to the individual maps. The diagram exhibits fluctuating peaks and dense bands, reflecting irregular dynamics and sensitivity to both r and the nested function structure. The combination increases entropy and decreases predictability which is ideal for securing cryptographic systems.

Small perturbations in parameters or initial values lead to significantly different outputs which is a desirable trait in chaos-based encryption.

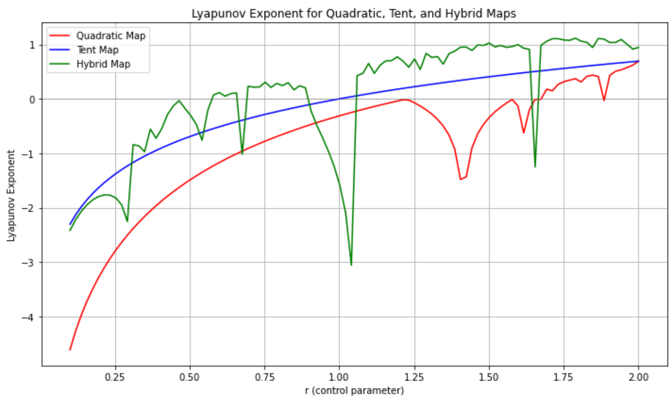

Figure 5.2: Lyapunov Exponent of Quadratic, Tent and Hybrid Maps

Lyapunov Exponent of the Quadratic Map

The Lyapunov exponent of the quadratic map, plotted across the control parameter range starts negative, indicating stable fixed points and becomes positive near r≈1.25, signifying the onset of chaos. The curve displays a non-monotonic behavior with alternating windows of positive and negative values, corresponding to periodic windows within the chaotic region.

The quadratic map transitions into chaos via period doubling, and the Lyapunov plot reflects this rich structure. However, the presence of regular patterns implies a partially predictable system, which could be a vulnerability in cryptographic contexts if not properly masked.

Lyapunov Exponent of the Tent Map

As shown in Figure 5.2, the Lyapunov exponent of the tent map remains positive for most values of r>1r and approaches a maximum near r=2. Unlike the quadratic map, the curve is smoother and consistently above zero, with minimal periodic interruptions.

This suggests that the tent map exhibits stronger and more persistent chaos, which makes it suitable for secure systems requiring high entropy and minimal long-term predictability. Its consistent positive Lyapunov exponent implies that its output diverges exponentially, maximizing uncertainty.

Lyapunov Exponent of the Hybrid Map

The Lyapunov exponent of the hybrid map (Figure 5.2) demonstrates a more complex and irregular profile. It becomes positive earlier than in the quadratic map and achieves higher peaks, indicating stronger chaotic behavior. The curve is rugged, with frequent fluctuations, suggesting intermittent chaos and sensitive dependence on both the control parameter and the composition of the mapping functions.

The hybrid system inherits and enhances the chaotic dynamics of both parent maps. The early onset and higher magnitude of positive Lyapunov exponents highlight its high sensitivity and randomness that are key indicators of cryptographic strength. Its irregular structure confirms dynamic richness, suitable for secure key generation and message scrambling.

5.3 Time Series Analysis

The time series diagram offers crucial insight into the temporal behavior of chaotic systems. It reveals how successive values evolve with respect to iteration steps, thereby illustrating the dynamical complexity and unpredictability of the system. Figure 5.3 displays the time series plots of the Quadratic Map, Tent Map, and the proposed Hybrid Map under chaotic regimes.

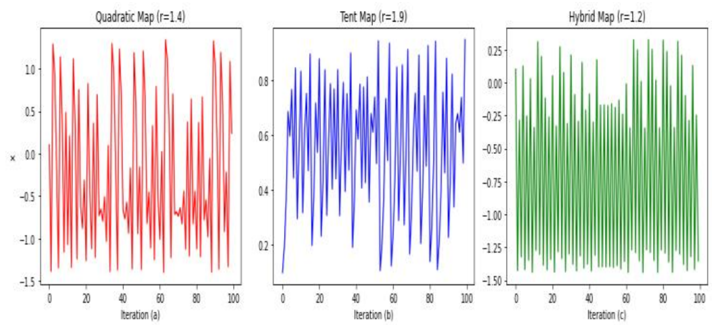

Figure 5.3: Time Series Analysis of Quadratic, Tent and Hybrid Map

Quadratic Map Time Series Analysis

The time series of the quadratic map (Figure 5.3a) for r=1.4 exhibits an irregular and non-repeating pattern. This non-periodic behavior is a characteristic of deterministic chaos. The values oscillate unpredictably, diverging from any fixed or periodic behavior. The sequence reveals a sensitive dependence on initial conditions, whereby a minute alteration in the starting point results in drastically different trajectories. Although the quadratic map does produce chaotic behavior, it may suffer from limitations such as finite state-space coverage and potential symmetry, which can be exploited in cryptanalysis.

Tent Map Time Series Analysis

In contrast, the tent map's time series for r=1.9 reveals a jagged, piecewise-linear dynamic, with values constrained within the unit interval [0,1]. In Figure 5.3b, the system alternates between upward and downward slopes, creating a visibly sharp yet chaotic trajectory. Despite its linearity in segments, the tent map remains chaotic and ergodic, and it demonstrates stronger statistical properties, including uniform distribution. This makes it a favorable candidate in pseudo-random number generation and stream cipher design. Its simplicity, however, may lead to predictability when subjected to exhaustive attack models if not hybridized or enhanced.

Hybrid Map Time Series Analysis

The time series of the hybrid quadratic-tent map (with r=1.2) as shown in Figure 5.3c, is a more sophisticated chaotic trajectory. It synthesizes the nonlinearity of the quadratic map with the piecewise behavior of the tent map. The resulting dynamics exhibit sharper fluctuations, complex structure, and an absence of repetition, indicating enhanced chaotic behavior. The combination introduces a richer state space, higher entropy, and increased randomness, addressing the individual limitations of both maps. This improved dynamical performance validates its suitability for secured cryptosystems, as it ensures unpredictability, increased key space, and resistance to statistical and brute-force attacks.

5.4 Permutation and Approximate Entropy Analysis

Entropy measures provide insight into the complexity and unpredictability of a time series. While Shannon entropy evaluates the statistical randomness of values, other entropy metrics like Permutation Entropy and Approximate Entropy assess the temporal structure and pattern diversity of chaotic systems. These are particularly important in cryptographic applications where both unpredictability and irregularity are desired. Figure 5.4 shows the Permutation and Approximate Entropies of Quadratic, Tent and Hybrid maps

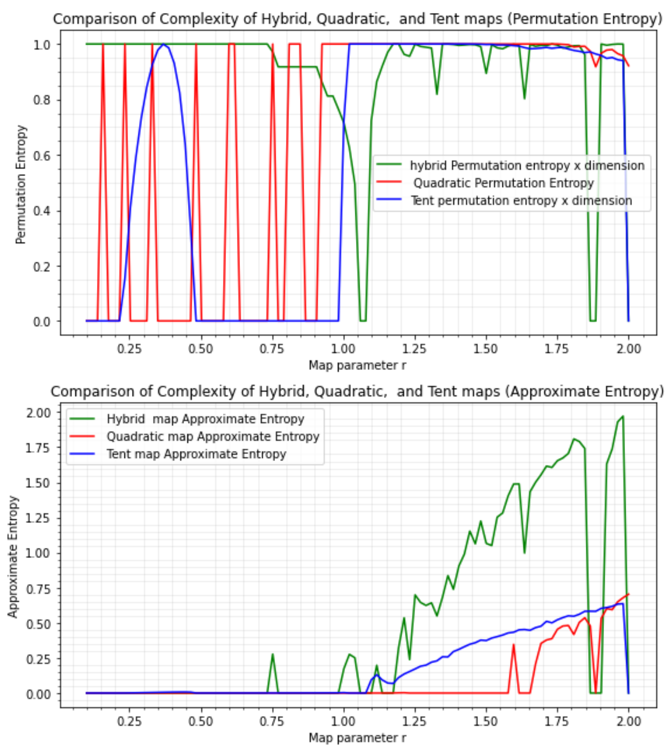

Figure 5.4: Permutation and Approximate Entropy

Permutation Entropy (PE) is a robust metric that evaluates the disorder in the ordering of time series values, rather than their magnitude. It ranges between 0 (completely predictable) and 1 (completely random), making it suitable for detecting subtle changes in dynamics.

Quadratic Map (PE)

The increase in PE across this transition confirms the onset of higher dynamical complexity and unpredictability. However, PE also fluctuates due to the presence of periodic windows amidst chaos.

Tent Map (PE)

In contrast to the quadratic map, the tent map maintains a consistently high PE across most values of r, except when r is less than 1, where the system is trivial. This reflects the inherently chaotic nature of the tent map, which lacks periodic windows and shows persistent structural disorder. Its linearity, combined with abrupt slope changes, produces maximal ordinal pattern diversity, thus elevating PE values.

Hybrid Map (PE)

The hybrid map constructed by feeding the quadratic map output into the tent map, exhibits high and relatively stable PE across a wide range of r. Unlike the quadratic map, it suppresses periodicity and reduces the fluctuation of entropy values. This stability indicates enhanced randomness and a more uniform chaotic regime, making it suitable for applications requiring robust entropy sources.

Approximate Entropy (ApEn) Analysis

Quadratic Map (ApEn)

ApEn displays a trend similar to PE, with low values in the regular regime and a sharp increase near the onset of chaos. The peak ApEn values are observed in regions where the system exhibits high sensitivity to initial conditions. However, due to its dependence on tolerance thresholds and pattern matching, ApEn can be affected by noise and finite data length.

Tent Map (ApEn)

The tent map produces high ApEn values consistently across r, indicating a high degree of pattern unpredictability. Compared to the quadratic map, its ApEn values are less sensitive to changes in the control parameter, demonstrating a more uniform complexity and randomness in the temporal patterns of the generated sequence.

Hybrid Map (ApEn)

From Figure 5.4, the hybrid map achieves the highest and most stable ApEn values among the three systems. The interaction between the nonlinear characteristics of the quadratic map and the piecewise nature of the tent map results in a complex attractor structure that reduces the recurrence of patterns. The enhanced irregularity of the time series confirms the hybrid map's ability to generate highly unpredictable sequences, a desired property in secure information systems.

6. CONCLUSION

This paper has demonstrated that the quadratic and tent maps provide an excellent foundation for secure cryptographic systems. Through bifurcation, Lyapunov exponent, time series analysis and entropy. We have shown that the hybrid of both maps possesses strong chaotic properties that are desirable in cryptographic applications.

The hybrid function provides enhanced randomness, greater entropy, and resistance to brute-force attacks by exploiting the nonlinearity and piecewise nature of its components. Implementations using this model are suitable for image encryption, key stream generation, and secure lightweight communication protocols.

Future work could extend to hardware implementation, testing resistance to known plaintext attacks, and exploring other entropy-enhancing chaotic hybrids.

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