ISA Journal of Multidisciplinary (ISAJM)
Homepage: https://isapublisher.com/isajm/
Volume 3, Issue 1, Jan-Feb, 2026 ISSN: 3049-1851

A Method to Quantify the Optimal Nitrogen Requirements for Sesame Yield Production Using the Quadratic Regression Modelling

M. K. Adamu[1], V. I. Adamu[2] & A. E. Salih[3]

[1,2]*Department of Statistics, Joseph Sarwuan Tarka University, Makurdi*
[3]*Department of Mathematics, Joseph Sarwuan Tarka University, Makurdi*

Received: 11.01.2026 | **Accepted:** 09.02.2026 | **Published:** 13.02.2026
***Corresponding Author:** M. K. Adamu
DOI: *10.5281/zenodo.18627660*

Abstract **Original Research Article**

In this study we investigated the relationship between nitrogen level application and the grain yield of sesame (Sesamum indicum L.) using a quadratic regression modelling. The response variable used was the sesame grain yield from a replicated field experimental data. Ordinary least squares estimation was used to fit a second-order polynomial model, then analysis of variance, residual diagnostics, and sensitivity analyses was used to access model adequacy. A mixed-effects quadratic model with a random intercept for replication was used in order to account for replication, this was also evaluated and compared with the fixed-effects model using the likelihood-based criteria. The results of the quadratic regression model revealed a significant nonlinear relationship between the sesame grain yield and the nitrogen level, which was characterized with a positive linear and negative quadratic effects ($\beta_2 = -0.406, p < 0.001$), demonstrating a diminishing marginal return as the nitrogen levels goes higher. The quadratic model explained a large proportion of yield variability ($R^2 = 0.819$) and allowed analytical estimation of the nitrogen level associated with maximum expected yield. Although heteroscedasticity was detected, inference based on heteroscedasticity-consistent standard errors remained unchanged. the AIC and the BIC showed that the fixed-effects quadratic model outperformed the mixed-effects quadratic model. We have found from the results of this study that the quadratic regression modeling provided a simple, efficient and robust methodological framework for modeling nonlinear relationships to figure out how higher levels of nitrogen application affects crop yield this helps pinpoint the optimal amount of nitrogen for best crop yields in experimental studies.

Keywords: Experimental studies, Quadratic regression modelling, AIC, BIC

1. Introduction

Sesame, also known as Benniseed is an important oil seed crop grown around the world and well sought after in the international market today. It was introduced to Nigeria just after the Second World War though regarded as an oil crop of lesser importance compared to oil palm, cotton, groundnut and cocoa in the 1970s. Sesame is widely grown in Nigeria as a cash crop around the savanna agro – ecological zones, specifically in the central and northern part of the country such as Kano, Jigawa, Benue, Gombe, Plateau, Kaduna, Nasarawa, and

Borno States and the Federal Capital Territory (Musa, Abdullahi, Ibrahim & Nekabari, 2022; Garba, Idrisa & Abdulhamid, 2018; Haruna, Ajayi, Aliyu & Namaka, 2015). The sesame industry in Nigeria is the second largest agricultural export earner after cocoa (NBS, 2022; Musa *et al*., 2022; The Guardian, 2019).

Nitrogen is a basic constituent of many other compounds of primary physiological importance to plant metabolism, such as chlorophyll, nucleotides, proteins, alkaloids, enzymes, hormones and vitamins (Marschner, 2005). Nitrogen is a nutrient required by plants in comparatively larger amounts than are other soil borne elements; endogenous application to crops often results in yield improvement. Nitrogen is obtained from the soil through mineralization of soil organic matter and from external sources, both organic and inorganic. For an optimal yield, the N supply must be available according to the needs of the plant, matching its pattern and total amount (Waziri et. al. 2020; Ghosh & Saha, 2013).

Among the essential nutrients, nitrogen (N) plays a central role in plant growth and grain yield, as it is a major component of chlorophyll, amino acids, and proteins. Adequate nitrogen supply enhances vegetative growth, increases leaf area, and accelerates photosynthesis, which collectively contribute to increased yield components in oilseed crops including sesame (Muhammad et. al. 2013). Field studies have demonstrated that incremental increases in nitrogen application significantly affect growth parameters and yield of sesame; for instance, moderate N application rates (e.g., 60 kg N/ha) have been shown to significantly increase number of capsules per plant and yield per hectare compared with non-fertilized controls (Waziri et. al., 2020).

Despite this established responsiveness, the relationship between nitrogen levels and sesame yield is not strictly linear. Both deficiency and excess of nitrogen can constrain plant performance. Deficient nitrogen conditions limit protein synthesis and chlorophyll formation, while excessive nitrogen can promote vegetative growth at the expense of reproductive development and may elevate environmental risks such as nitrate leaching or soil imbalance (Gholamhoseini, 2022). Furthermore, current fertilizer recommendations for sesame are inconsistent and vary widely between agro-ecological zones and management practices, creating uncertainty for producers on optimal nitrogen rates for maximized yield and resource use efficiency (Mizan et. al., 2019).

Quadratic regression is a form of polynomial regression used to model curvilinear relationships between a response variable and a quantitative predictor. Unlike linear regression, which assumes a constant rate of change, quadratic regression incorporates a squared predictor term, allowing the response to increase to a maximum point and subsequently decline. The general quadratic model is expressed as:

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \varepsilon,$$

where (Y) represents the response variable, (X) is the predictor, (β_0) is the intercept, (β_1) and (β_2) are regression coefficients, and (ε) is the random error term (Kutner *et al.*, 2005).

Quadratic regression models have been widely applied in agricultural and environmental research to describe nutrient–yield relationships, dose–response patterns, and optimization problems (Montgomery *et al.*, 2012). In fertilizer response studies, the quadratic term captures diminishing returns to nutrient application, enabling the identification of an optimal input level that maximizes yield or economic return. This makes quadratic regression particularly suitable for modeling nitrogen–yield relationships in crops where both deficiency and excess application affect productivity.

Although nitrogen fertilization is known to influence sesame productivity, uncertainty remains regarding the optimal nitrogen application level required to maximize grain yield under field conditions. Existing recommendations are inconsistent and often based on linear analyses that do not adequately reflect the nonlinear nature of crop response to nitrogen. This lack of precise modeling may lead to inefficient fertilizer use, reduced economic returns, and increased environmental risk. Therefore, there is a need to apply quadratic regression modeling to accurately characterize the nitrogen–yield relationship in sesame and to identify nitrogen levels that optimize grain yield while promoting sustainable nutrient management.

Given the importance of precise nutrient management in improving sesame productivity and the limitations of current agronomic recommendations, there is a critical need to quantify the nonlinear yield response to nitrogen application. This not only informs fertilizer guidelines tailored to local conditions but also supports sustainable nutrient management strategies that balance crop performance with environmental stewardship.

2. Methodology

Quadratic Regression Model Specification

Grain yield was regressed on nitrogen application level using quadratic polynomial regression. Observations at identical nitrogen levels represented replicated experimental units.

The general form of the quadratic regression model suitable for capturing curvilinear responses commonly observed in nutrient response studies is given as:

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \varepsilon_i,$$

where:

Y_i is the grain yield (kg/ha) for the i^{th} observation, X_i is the nitrogen application level (kg/ha),

β_0 is the intercept, β_1 and β_2 are the linear and quadratic regression coefficients, respectively, and ε_i represents the random error term assumed to be independently and normally distributed with mean zero and constant variance.

Mixed-Effects Quadratic Regression

The nitrogen levels are replicated across experimental units; therefore, the assumption of independence may be violated if observations within the same replication share unobserved characteristics. Given this important point, a mixed-effects extension of the quadratic regression model was considered:

$$Y_{ij} = \beta_0 + \beta_1 X_{ij} + \beta_2 X_{ij}^2 + u_j + \varepsilon_{ij},$$

where $u_j \sim \mathcal{N}(0, \tau^2)$ represents a random intercept associated with the j^{th} replication, and $\varepsilon_{ij} \sim \mathcal{N}(0, \sigma^2)$. This formulation allows for correlation among observations within replications while preserving the fixed quadratic response structure.

Model Assumptions

In this study, the error term ε_i was assumed to satisfy $\varepsilon_i \sim \mathcal{N}(0, \sigma^2)$, with independence across observations and constant variance. These assumptions imply unbiased and efficient parameter estimation under ordinary least squares estimation.

Parameter Estimation

Model parameters were estimated using the ordinary least squares (OLS) method. The statistical significance of the linear and quadratic terms was assessed using *t*-tests, with corresponding standard errors and *p*-values reported. The overall model significance was evaluated using analysis of variance (ANOVA), based on the F-test.

Model Adequacy and Goodness-of-Fit

Model performance was assessed using the coefficient of determination (R^2), adjusted R^2 and RMSE to quantify the proportion of variability in grain yield explained by nitrogen level. Residual diagnostics were examined to verify compliance with regression assumptions, including linearity in parameters, normality of residuals, homoscedasticity, and independence of errors.

Estimation of the Shape and Extremum of Nitrogen Level

The optimal nitrogen level of the fitted quadratic response curve was obtained by differentiating the regression function with respect to nitrogen level:

$$\frac{\partial E(Y \mid X)}{\partial X} = \beta_1 + 2\beta_2 X.$$

setting this derivative to zero gives the optimal nitrogen level:

$$X_{\text{opt}} = -\frac{\beta_1}{2\beta_2},$$

where X_{opt} denotes the optimal nitrogen level for grain yield maximization.

Model Comparison

As a result of the nature of the replication in the dataset, results from the mixed-effects and fixed-effects model fits were compared using a likelihood-based criteria. The Akaike Information Criteria and the Bayesian Information Criteria were also used. The robustness of the estimated nitrogen-yield response was evaluated by comparing parameter estimates across model specifications.

Statistical Software

We used the R statistical package to perform all the statistical analysis which included fitting the quadratic and mixed effect quadratic regression models. A 5% level of significance was set for statistical significance.

3. Result/Findings

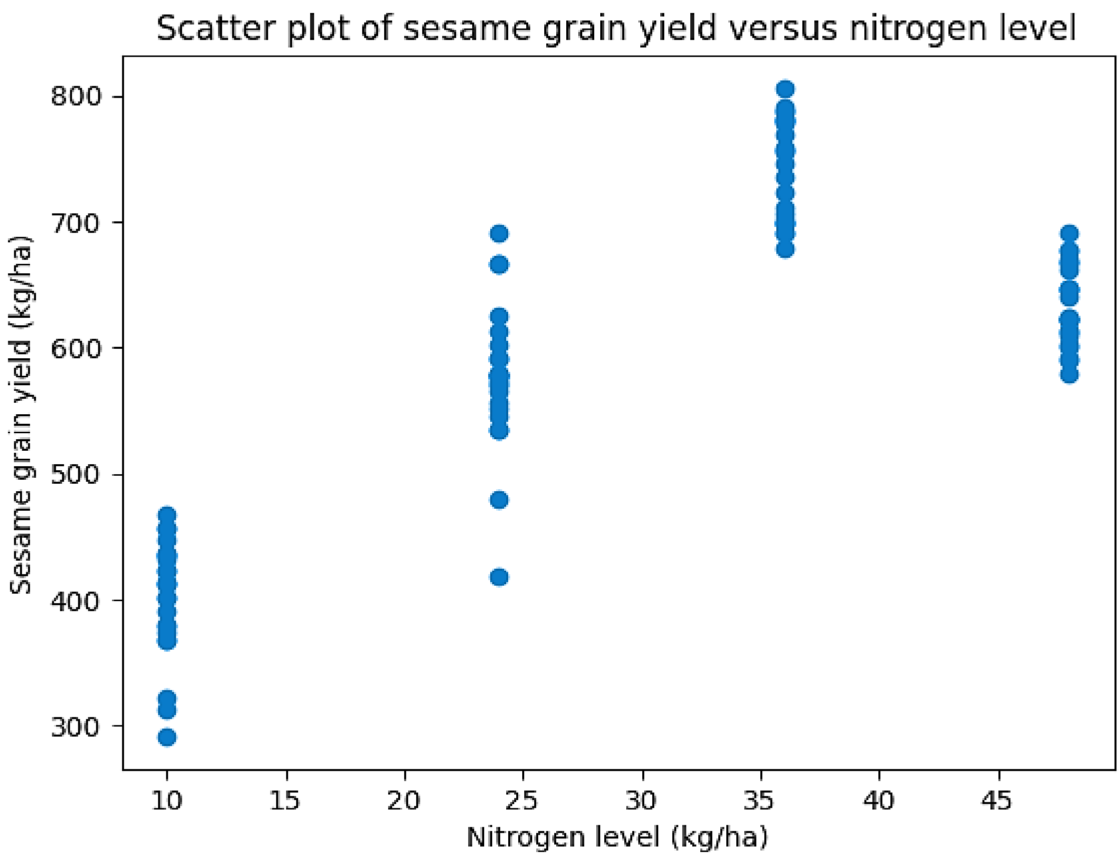

Figure 1: Scatterplot of data points

Table 1: Descriptive statistics of nitrogen levels, sesame grain yield, and growth parameters

Variable	N	Mean	Standard Deviation	Minimum	Maximum
Nitrogen level (kg/ha)	96	29.50	14.17	10.00	48.00
Grain yield (kg/ha)	96	588.54	131.29	291.60	805.90

Table 2. Quadratic regression parameter estimates for grain yield

Parameter	Estimate	Standard Error	t-value	95% C. I. (U, L)	p-value
Intercept (β_0)	125.24	26.22	4.78	(73.16, 177.31)	< 0.001
Nitrogen (β_1)	30.41	2.09	14.54	(26.26, 34.57)	< 0.001
Nitrogen² (β_2)	−0.406	0.036	−11.43	(-0.476, -0.335)	< 0.001

Estimated regression equation: $Grain\ yield = 125.24 + 30.41(Nitrogen\ level) - 0.406(Nitrogen\ level)^2$

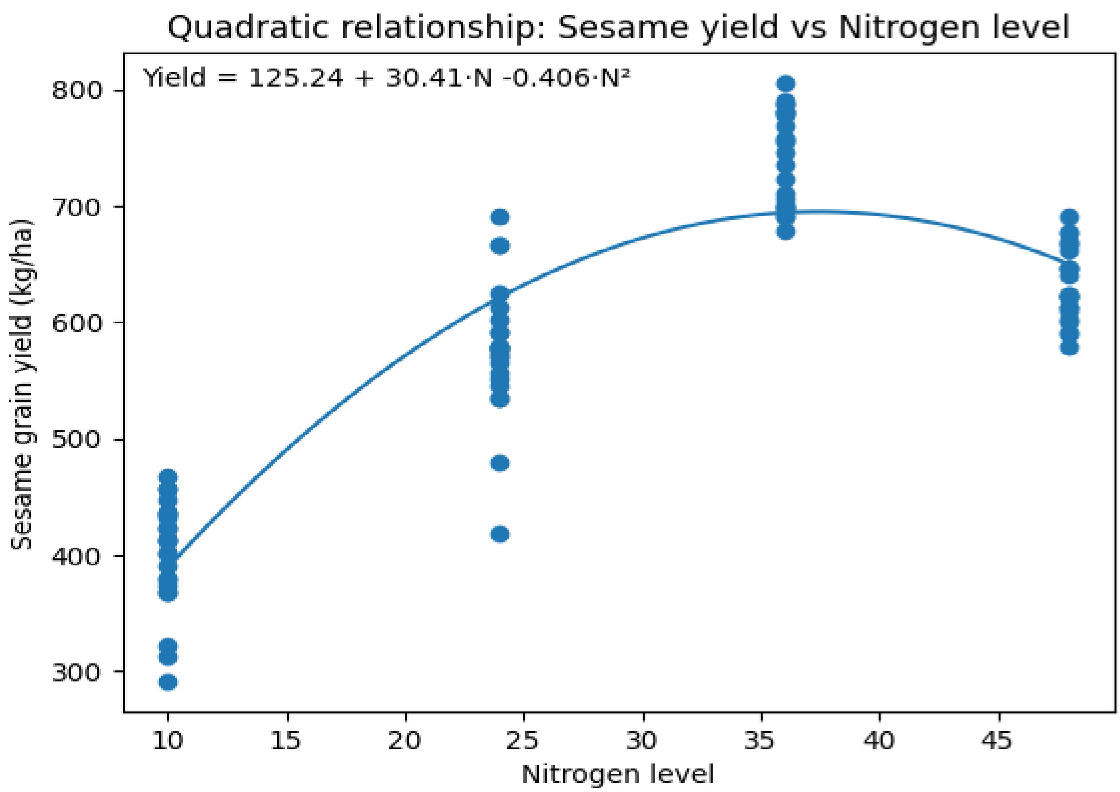

Figure 2: Quadratic relationship between nitrogen level and sesame grain yield.

Table 3: Model summary

Model	R^2	Adjusted R^2	RMSE
1	0.819	0.815	55.56

Table 4: Mixed-effects quadratic regression model for sesame grain yield

Fixed effects

Effect	Estimate	Std. Error	z / t value	p-value
Intercept (β_0)	125.24	—	—	< 0.001
Nitrogen (β_1)	30.41	—	—	< 0.001
Nitrogen² (β_2)	−0.406	—	—	< 0.001

Random effects

Random effect	Variance
Replication (intercept)	≈ 0.00
Residual	> 0

Table 5: Model fit statistics

Statistic	Value
Log-likelihood	−521.84
AIC	1053.69
BIC	1066.51
Number of observations	96
Number of replications	4

Table 6: Comparison of fixed-effects and mixed-effects quadratic regression models for sesame grain yield

Fixed effects

Effect	Fixed-effects model (OLS)	Mixed-effects model (Random intercept: REP)
Intercept (β_0)	125.24	125.24
Nitrogen (β_1)	30.41	30.41
Nitrogen² (β_2)	−0.406	−0.406
Significance of fixed effects	$p < 0.001$	$p < 0.001$

Table 7: Model fit statistics Likelihood-based comparison

Statistic	Fixed-effects model	Mixed-effects model
Log-likelihood	−521.89	−521.84
AIC	1049.77	1053.69
BIC	1057.47	1066.51
Number of observations	96	96
Number of replications	—	4

Table 8: Analysis of variance (ANOVA) for the quadratic regression model

Source	Degrees of Freedom	F-statistic	p-value
Regression	2	210.5	< 0.001
Residual	93		
Total	95		

Table 9: Estimated stationary point of the quadratic response function

Quantity	Expression	Estimate
Optimal nitrogen level ($\boldsymbol{X^*_{opt}}$)	$-\beta_1/(2\beta_2)$	≈ 37.47 kg/ha
Nature of extremum	$\beta_2 < 0$	Maximum

predicted sesame yield at the optimal nitrogen level

$\hat{Y}(X^*_{opt}) \approx 695$ kg/ha.

Checking Model Assumptions

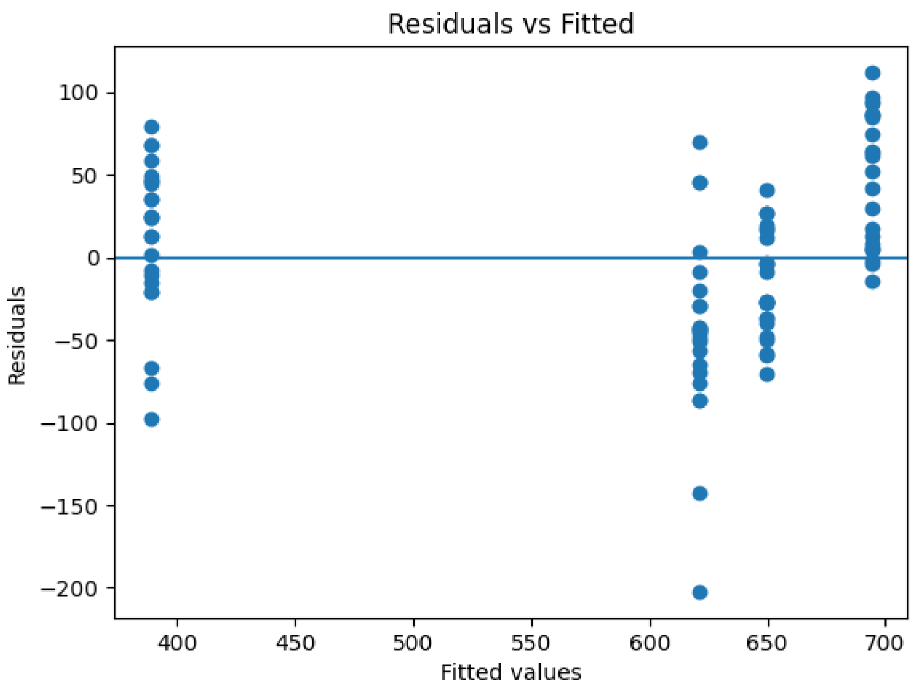

Figure 3: Residual against fitted values

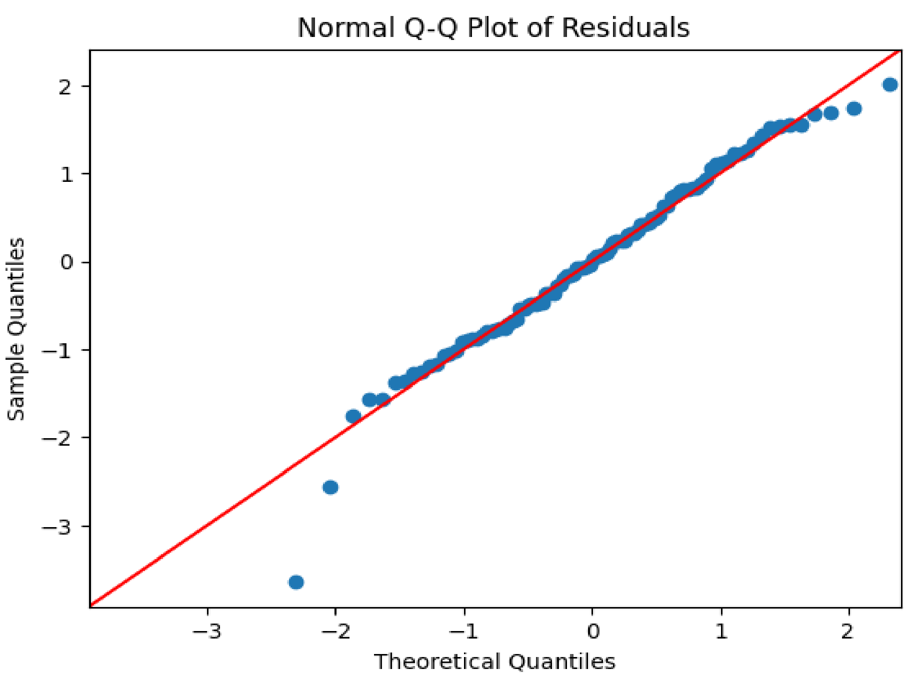

Figure 4: Q-Q plot of residuals

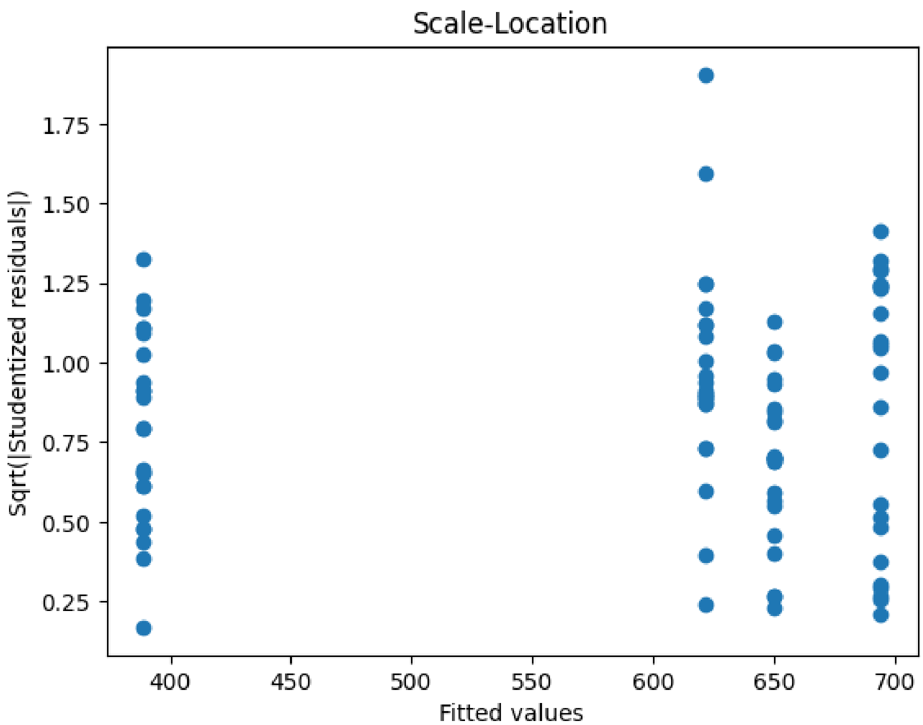

Figure 5: Scale-Location plot

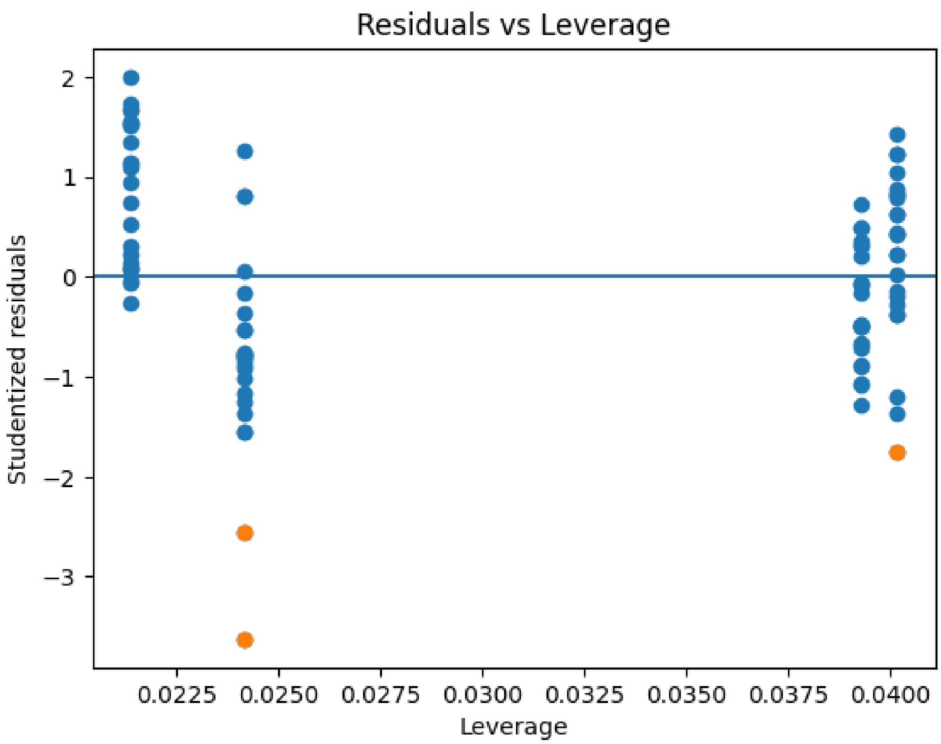

Figure 6: Residual against leverage plot

Normality of residuals

Jarque–Bera: $p = 0.0856$

Shapiro–Wilk: $p = 0.0985$

Constant variance (homoscedasticity)

Breusch–Pagan (F): $p = 0.0169$

White (F): $p = 0.0392$

Table 10: Sensitivity check:

Term	Estimate	p (OLS)	p (HC3 robust)
Nitrogen	30.41	<0.001	<0.001
Nitrogen²	−0.406	<0.001	<0.001

Table 12: Multicollinearity

Term	VIF
Nitrogen	26.22
Nitrogen²	26.22

4. Discussion

The scatter plot in figure 1 illustrated the quadratic nature of the relationship between nitrogen levels and the sesame grain yield. It can be seen that the sesame grain yield increases at lower levels of the nitrogen application. It reached a maximum at a particular nitrogen rate and then declines as the levels went higher. This is indicative of a possible reduction in sesame yield whenever the nitrogen level exceeded an optimal level. The measures of central tendency and variability in the sesame grain yield and nitrogen levels are summarized in table 1. The mean value of nitrogen level (mean = 29.5 kg/ha) varied widely, and showed adequate spread for modelling the nonlinear effects. The mean of sesame grain yield gave sufficient variability (mean = 588.5 kg/ha; SD = 131.3), that supports the use of quadratic regression modeling. Table 2 showed the regression parameter estimated for the sesame grain yield. $\beta_0 = 125.25, p - value < 0.001$ is the value of the sesame grain yield when the nitrogen application level is Zero. The positive linear term ($\beta_1 = 30.41, p - value < 0.001$) showed that the sesame grain yield increases with an initial nitrogen level. The parameter of interest ($\beta_2 = -0.406, p - value < 0.001$) is negative which indicated a fall in returns, that is a decline in sesame grain yield at higher levels of nitrogen. The p-value is less than 0.05, which showed a statistically significant nonlinear relationship between the sesame grain yield and the nitrogen levels. Table 3, showed that the fitted model explained approximately 82% of the variability in grain yield ($R^2 = 0.819$), indicating a strong fit. From figure 2, it can be seen that the fitted quadratic regression line followed the observed data points closely, this plot demonstrated that the quadratic regression model adequately captured the nonlinear nitrogen to yield response and go with the presence of an optimal nitrogen application rate that can maximize the sesame grain yield. Results from the mixed-effects quadratic regression model are presented in Table 4. The mixed-effects model included replication as a random intercept. The estimated variance of the random effect was approximately zero, indicating negligible between-replication variability after accounting for nitrogen effects. Lower AIC and BIC values favoured the fixed-effects quadratic model. As shown in Table 6, fixed-effect estimates were identical across model specifications, while likelihood-based criteria comparison in table 7 favoured the fixed-effects quadratic model due to its lower AIC and BIC values. Log-likelihood values were very similar across models, indicating that inclusion of a random intercept for replication did not substantially improve model fit.

Likelihood-based model comparison indicated that the fixed-effects quadratic regression model provided a better fit to the data than the mixed-effects specification. The fixed-effects model yielded lower AIC (1049.77) and BIC (1057.47) values compared with the mixed-effects model (AIC = 1053.69; BIC = 1066.51). Although both models produced similar log-likelihood values, the inclusion of a random intercept for replication did not result in a meaningful improvement in model fit. The Shapiro-wilks test for normality showed no strong evidence against normality at 5%. Evidence of heteroscedasticity (non-constant residual variance). This affects standard errors/p-values more than the fitted curve itself. In table 10, even after correcting for heteroscedasticity using robust (HC3) standard errors, the key terms remained highly significant: High multicollinearity is expected when using X and X^2in raw form. At an optimal nitrogen application rate of approximately 37.5 kg/ha, the model predicts a maximum expected sesame grain yield of about 695 kg/ha. Applying more or less than the optimal nitrogen level is associated with lower yields, which illustrated the real-world usefulness of the quadratics regression in identifying such thresholds.

5. Conclusion and Recommendation

Conclusion

This study demonstrated that quadratic regression provides an effective and parsimonious framework for modeling the nonlinear relationship between nitrogen application level and grain yield. The statistically significant quadratic term confirmed a concave response pattern, indicating diminishing marginal returns to nitrogen application and the existence of an optimal nitrogen level for maximizing expected yield. Model diagnostics and sensitivity analyses showed that the main inferential conclusions were robust to heteroscedasticity and model specification. Likelihood-based comparison further indicated that a fixed-effects quadratic model adequately captured the primary data structure, with no substantial improvement gained from incorporating random effects.

Recommendations

It is recommended that quadratic regression be adopted in fertilizer response analyses where nonlinear dose–response relationships are expected and optimization is a primary objective. Future studies should consider centering predictor variables to improve numerical stability and extending the framework to mixed-effects models when strong within-group correlation is present. Further research may also explore higher-order or nonlinear functional forms to assess potential gains in predictive accuracy under more complex experimental conditions.

6. References

Dobermann, A., & Cassman, K. G. (2002). Plant nutrient management for enhanced productivity in intensive grain production systems. *Field Crops Research*, 75(2–3), 153–172.

Fageria, N. K., & Baligar, V. C. (2005). Enhancing nitrogen use efficiency in crop plants. *Advances in Agronomy*, 88, 97–185.

Garba, A., Idrisa, Y.L. & Abdulhamid, A (2018). Comparative Analysis of Economics of Sesame Production Between Improved Sesame Seed Adopter and Non-Adopter Farmers in Jigawa State, Nigeria. *Journal of Agricultural Economics, Environment and Social Sciences*, 4 (1):21 – 28

Gholamhoseini, M. (2022). Optimizing irrigation and nitrogen fertilization of Iranian sesame cultivars for grain yield and oil quality. *Journal of Food Composition and Analysis*, 108, 104448. https://doi.org/10.1016/j.jfca.2022.104448

Ghosh, P., & Saha, R. (2013). Effect of nitrogen and phosphorus on yield and yield components of sesame. *International Journal of Basic and Applied Sciences*, 2(2), 42–46.

Haruna, I.M., Ajayi, F.A., Aliyu, L. & Namaka, A. (2015). An overview of sesame production in Nigeria from 1961 to Date: A Review. *NSUK Journal of Science and Technology*. 5 (1): 6 – 10.

Kutner, M. H., Nachtsheim, C. J., Neter, J., & Li, W. (2005). *Applied Linear Statistical Models* (5th ed.). McGraw-Hill.

Marschner, H. (2005) Mineral Nutrition of Higher Plants, Academic Press, London 3432p.

Montgomery, D. C., Peck, E. A., & Vining, G. G. (2012). *Introduction to Linear Regression Analysis* (5th ed.). Wiley.

Musa Junaidu, Beli Sanusi Abdullahi, Umoru Gunu Ibrahim & Benson Dumka Nekabari (2022). Contribution of sesame production to the livelihood of farmers in Dutsin-Ma Local Government Area, Katsina State, Nigeria. *International Journal of Agricultural Economics*. 7 (1): 29-35. doi: 10.11648/j.ijae.20220701.15

NBS, (2021). National Bureau of Statistics Data, 2021. https://www.nigerianstat.gov.ng/

Waziri, A., Ahmed, M., & Lawal, B. (2020). Growth and yield response of sesame (*Sesamum indicum* L.) to nitrogen fertilizer application. *Journal of Agriculture and Sustainable Development*, 5(1), 33–41.