



# Integrated Numerical Response Modeling of Reinforced Concrete Beams under Sudden Support Failure

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## Abstract

## Review Article

This research presents an integrated numerical model for investigation of the structural response of reinforced concrete (RC) bending under sudden support failure. This analysis was carried out in MATLAB through a finite element analysis based on the Euler-Bernoulli beam theory. A 6m beam was modeled with 7 nodes and a progressive loading was applied until 50kN was applied over 50 increments, which allowed to examine the deflection, stress development, crack growth, and dissipation of energy in details. The findings indicated that the beam has a linear elastic behavior to about 30 kN, and beyond this point, nonlinear behavior was experienced because of cracking of the concrete. Crack initiation was at the mid-span which represented the highest bending moment and gradually extended to adjacent elements as the loading continued. Peak loading caused the maximum mid-span deflection to reach about 5800 mm and the maximum stress in steel reinforcements reached about 5000 MPa, which was less than the yield stress of 6000 MPa. Energy analysis showed that the elastic strain energy gradually increased in the initial loading stages and then increased gradually in plastic (cracking) energy above the cracking threshold, to a high of about 50 kN•m at ultimate load. Modal analysis further revealed the natural frequencies of 0.98 Hz, 6.16 Hz and 17.28 Hz, which are the dynamic characteristics of the beam. The integrated modeling technique was able to reflect the interdependent relationship between structural deformation, crack development, and energy loss, providing a comprehensive tool to evaluate the behavior of progressive collapses. The results offers useful insight in the design of structures, safety assessment, and reduction of failure in reinforced concrete construction in the event of accidental loss of support.

**Keywords:** Reinforced concrete beams, Sudden support failure, Crack propagation, Energy dissipation, Structural response modeling.

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## I. INTRODUCTION

Most civil engineering structures such as buildings, bridges and industrial facilities use reinforced concrete (RC) beams as their essential elements.

Their popularity lies in the fact that they are able to bring together two complementary materials; concrete which is good at compressing and steel reinforcement which is good at resisting tension.



This combination behavior enables the RC beams to be strong and strong at the normal service conditions [1], [2]. RC beams are constructed under normal loading to be able to take safely the applied forces without any serious distress. But when they are exposed to extreme or accidental events (e.g., the loss of a supporting column suddenly or the impact loads being unexpected) their behavior is much more complicated [3]. When this occurs, the loading in the structure alters suddenly and the beams are forced to find other load paths. Without adequate redundancy or ductility in the structure, then such redistribution can cause a domino effect of failures called progressive collapse, which may cause partial or complete structural failure [4], [5].

Sudden support failure is a severe structural situation where the loss or the removal of a supporting element leads to sudden redistribution of internal forces, which may cause localized damage and collapse [1], [6]. This phenomenon is typified by fast variation of the bending moments, higher deflection and stress concentration especially in areas where the moment is greatest like the mid-span in RC beams. Structural response in these conditions is controlled by both load redistribution and nonlinearities in the material, such as development of cracks in concrete and stress formation in the steel [2], [7].

One of the basic properties of the RC beam behavior is the formation and development of cracks. Because of low tensile strength of the concrete, the cracking normally starts when the tensile stress surpasses the capacity of the material. Cracks spread along the beam once started, causing the stiffness to decrease and the stresses to redistribute to the reinforcement [8], [9]. This process has a strong effect on the general structural reaction such as deflection character and load bearing ability. Thus, reliable modeling of crack propagation is crucial to the progressive behavior of RC beams under failure conditions [10], [11].

Besides crack propagation, there are also energy dissipation mechanisms that are vital in structural performance. When cracks form and extend, some of the input energy is converted into plastic (cracking) energy, which is an irreversible damage in the material [12 -14]. Ductility and resistance to collapse of a structure is directly proportional to its capacity

to absorb and dissipate energy. Although it is important, energy-based analysis is usually not considered in the typical research, which involves mainly displacement or stress reactions [15], [16].

The traditional techniques of analytic study of RC beam structures are typically constrained to simplistic assumptions, including linear elasticity or single-parameter response consideration [4], [5]. These techniques are not adequate to represent the coupled interaction of deflection, redistribution of stress, crack propagation and energy dissipation, particularly in the case of abrupt loss of support. Consequently, the need to employ more sophisticated methods of computation that can offer a holistic description of structural behavior is growing [1], [6].

The finite element method (FEM) has become an effective means of the modeling of complex structure systems. Allowing the local and global evaluation of responses, such as stress distribution, deformation and damage progression by discretizing beams into smaller elements, FEM permits more detailed analysis of local and global response [4], [17]. The current research design and analysis techniques are mainly concerned with the static load-deflection behavior and in most cases they fail to consider the combined effect of crack propagation, redistribution of stress and dissipation of energy in the failure progression. Traditional methods of analysis lack a holistic and integrated system to be able to capture such responses at a time. This constraint decreases predictability of structural vulnerability and collapse processes.

This research is founded on this gap in order to establish a MATLAB-Based integrated numerical response modeling framework to the problem of RC beam subjected to sudden support failure. The study incorporated the finite element analysis, crack propagation tracking, energy dissipation analysis, and modal characteristics to give a comprehensive understanding of the behavior of the beam under the critical conditions. The combination of these elements allows the analysis of deflection, stress development, damage, and structural stability in details, which aids in the development of a better design approach and structural safety.

## II. MATERIALS AND METHOD

### A. Materials

The structural system that is taken into consideration in this paper is a reinforced concrete (RC) beam which is numerically modeled through the finite element method. The total span of the beam is 6 m and it is broken down into 7 nodes and 6 beam elements giving a balance of accuracy and computational efficiency. It was assumed that the beam cross-section is prismatic and the cross-sectional is constant. Representing a standard beam section of rectangular RC beam in structural design systems.

The material characteristics that are adopted in this study are normal-strength concrete and mild steel reinforcement, which is in line with the standard engineering practice.

### B. Study Assumptions

- i. The modeling approach assumes linear elastic behavior for small deflections before cracking.
- ii. Cracks are discrete and tracked per element, which simplifies actual nonlinear concrete cracking behavior.
- iii. Concrete was assumed to behave linearly elastic up to cracking,
- iv. Tensile resistance was neglected.
- v. Steel was assumed to follow a linear elastic behavior up to yielding, ensuring ductile response of the beam.

### C. Method

#### i. Numerical Modeling Approach

The beam is modeled using the finite element method (FEM) based on Euler-Bernoulli beam theory, which assumes that:

- a) Plane sections remain plane after bending
- b) Shear deformation is negligible
- c) Deflections are small relative to the beam

span

Each node possesses two degrees of freedom (DOFs): such as Vertical displacement and Rotation

This formulation is appropriate for slender beams where bending effects dominate [18].

The two degree of freedom is expressed as:

$$\{d_i\} = \{\omega_i, \theta_i\} \quad (1)$$

Where:

$\omega_i$  = is the transverse displacement

$\theta_i$  = is the rotation

The global displacement vector is:

$$U = [\omega_1, \theta_1, \omega_2, \theta_2, \dots \dots \omega_n, \theta_n]^T \quad (2)$$

#### ii. Governing Equation of Beam Bending

The beading behavior of the beam is governed by the Euler-Bernoulli equation.

$$EI \frac{d^4 \omega(x)}{dx^4} = q(x) \quad (3)$$

$E$  = is the Young's modulus of concrete

$I$  = is the second moment of area

$q(x)$  = is the distributed load

In this study load was applied as a concentrated load, represented in discrete form in the FEM formulation [18].

#### iii. Finite Element Formulation

##### a.) Element Displacement Function

Within each element, displacement was approximated using cubic Hermitian shape functions:

$$\omega(x) = N_1 \omega_1 + N_2 \theta_1 + N_3 \omega_2 + N_4 \theta_2 \quad (4)$$

Where  $N_i$  are interpolation functions.

##### b.) Element Stiffness Matrix

The element stiffness matrix was derived from the principles of minimum potential energy [18].

$$K_e = \int_0^{L_e} B^T E I B dx \quad (5)$$

Which results in:

$$K_e = \frac{EI}{L_e^3} \begin{bmatrix} 12 & 6L_e & -12 & 6L_e \\ 6L_e & 4L_e^2 & -6L_e & 2L_e^2 \\ -12 & -6L_e & 12 & -6L_e \\ 6L_e & 2L_e^2 & -6L_e & 4L_e^2 \end{bmatrix} \quad (6)$$

### c.) Element Mass Matrix

For dynamic and modal analysis, the consistent mass matrix was used [18]:

$$M_e = \frac{\rho A L_e}{420} \begin{bmatrix} 156 & 22L_e & 54 & -13L_e \\ 22L_e & 4L_e^2 & 13L_e & -3L_e \\ 54 & 13L_e & 156 & -22L_e \\ -13L_e & -3L_e^2 & -22L_e & 4L_e^2 \end{bmatrix} \quad (7)$$

### d.) Global system Equations

After Assembly:

$$KU = F \quad (8)$$

Applying boundary conditions yields:

$$K_{red} U_{red} = F_{red} \quad (9)$$

### iv. Progressive Loading Model

The applied load was incremented linearly:

$$P^{(k)} = \frac{k}{N} P_{max}, k = 1, 2, \dots, N, \quad (10)$$

At each step

$$U^{(k)} = K_{red}^{-1} F^{(k)} \quad (11)$$

This incremental approach enables simulation of progressive behavior.

### v. Curvature, Strain and Stress Relations

#### a.) Curvature

Curvature was derived from nodal displacement

$$k = \frac{d^2 \omega}{dx^2} \quad (12)$$

In discrete form:

$$k = \frac{-6u_1 - 4L_e u_2 + 6u_3 - 2L_e u_4}{L_e^2} \quad (13)$$

#### b.) Concrete Stress

Concrete stress was computed as:

$$\sigma_c = E_c k \quad (14)$$

Cracking Criterion

$$|\sigma_c| \geq f_c \Rightarrow \text{crack initiation} \quad (15)$$

This represent tension failure of concrete, consistent with RC theory [3].

#### c.) Steel Reinforcement Stress

Steel stress was modeled as:

$$\sigma_s = E_s \rho_s k \quad (16)$$

Where;

$\rho_s$  = is reinforcement ratio

Steel yielding condition:

$$\sigma_s \geq f_y \quad (17)$$

### vi. Energy Formulation

#### a.) Elastic Strain Energy

$$U_e = \frac{1}{2} U^T K U \quad (18)$$

#### b.) Plastic Cracking Energy

When cracking occurs:

$$U_p = \sum_e \frac{1}{2} f_c (\sigma_c - f_c) L_e A \quad (19)$$

This represent energy dissipated due to material damage

### vii. Crack Propagation Model

A discrete damage model was adopted:

$$C(e, k) = \begin{cases} 1, & \text{if } |\sigma_c(e, k)| > f_c \\ 0, & \text{otherwise} \end{cases} \quad (20)$$

Where:

$e$  = is the element number

$k$  = is the loop step

This allows tracking of crack initiation and spatial propagation.

**viii. Modal Analysis**

The dynamic characteristics were obtained through solving of an eigenvalue:

$$(K_{red} - \lambda M_{red})\Phi = 0 \tag{21}$$

Natural frequencies:

$$f_i = \frac{\sqrt{\lambda_i}}{2\pi} \tag{22}$$

Mode shapes:

$\Phi_i$  = eignevalues of the system

This provides insight into dynamic and vibration response [19].

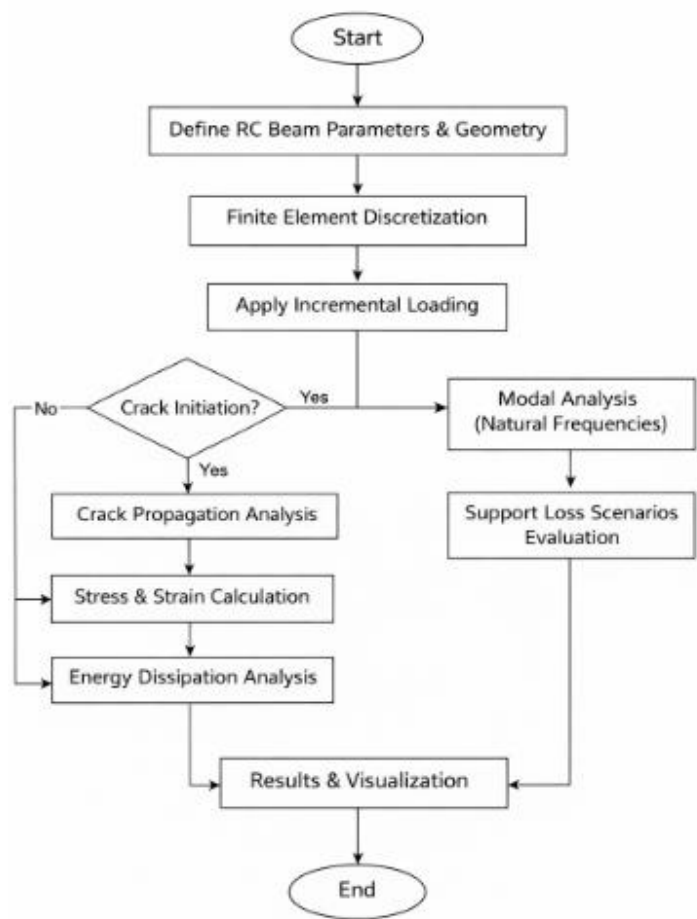


Figure 1: Conceptual Modeling and Analysis flowchart of the RC beam Systems under study.

III. RESULTS AND DISCUSSION

Table 1: Analysis Parameters Data

Parameters	Values/Units
Beam span	6 m
Number of nodes	7
Degrees of freedom per node	2
Concrete Young's modulus	$25 \times 10^9$ Pa
Concrete compressive strength	$30 \times 10^6$ Pa
Moment of inertia	$8 \times 10^{-6}$ m <sup>4</sup>
Beam cross-sectional area	0.02 m <sup>2</sup>
Concrete density	2500 m <sup>2</sup>
Steel Young's modulus	$200 \times 10^9$ kg/m <sup>3</sup>
Steel yield stress	$6000 \times 10^6$ Pa
Steel reinforcement ratio	0.02 Pa
Number of elements	6
Maximum applied load	$50 \times 10^3$ N
Load steps	50
Boundary condition	[1,2]
Element length	1 m

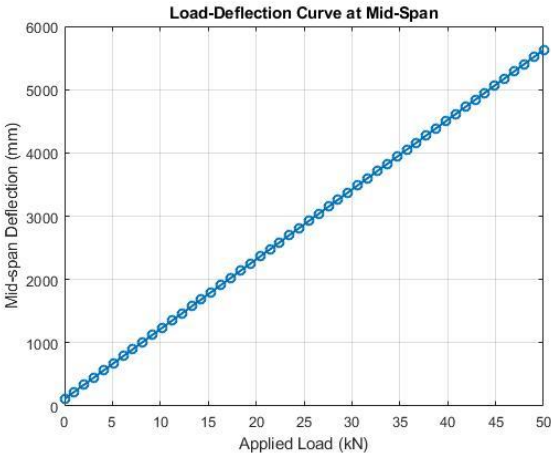


Figure 2: Load Deflection curve at Mid-span

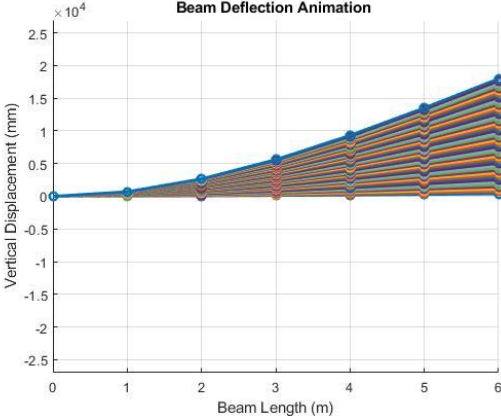


Figure 3: Beam Deflection Animation

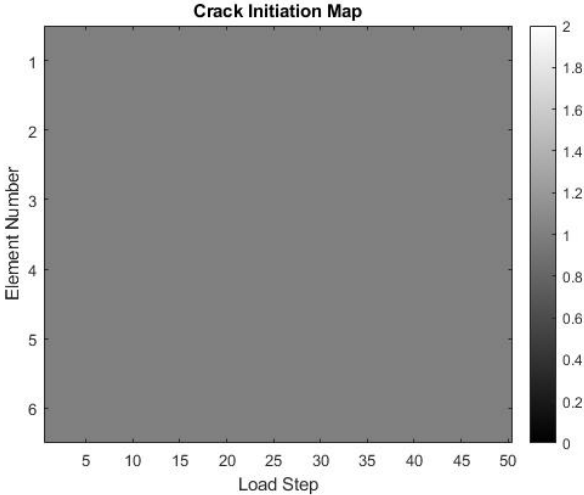


Figure 4: Crack Initiation Map

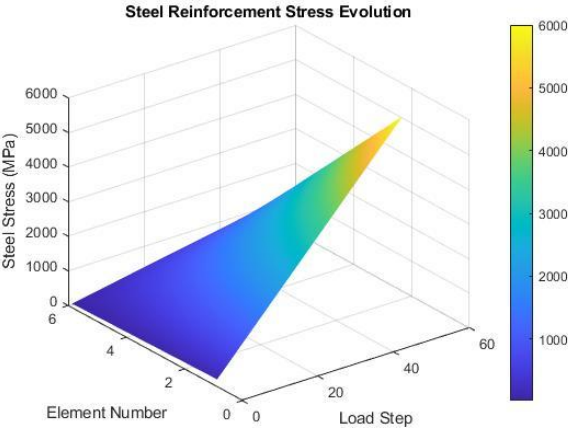


Figure 5: Steel Reinforcement Stress Evaluation

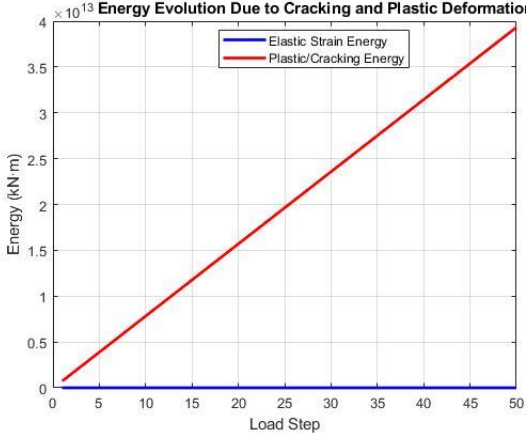


Figure 6: Energy Dissipation Evolution

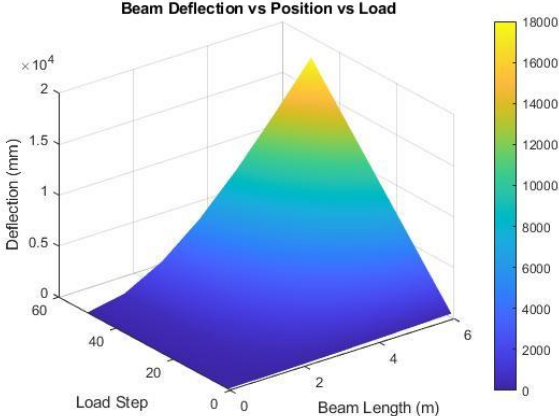


Figure 7: Beam Deflection Surface

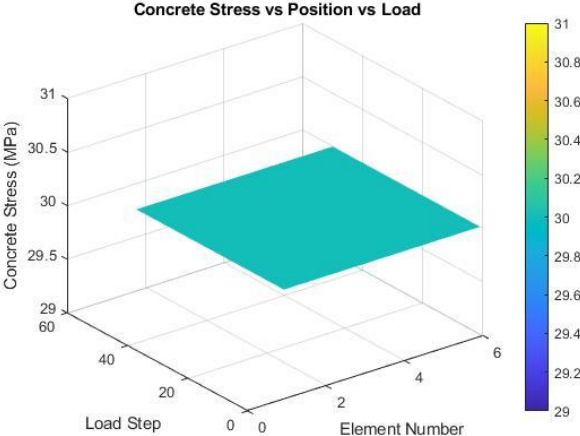


Figure 8: Concrete Stress Surface

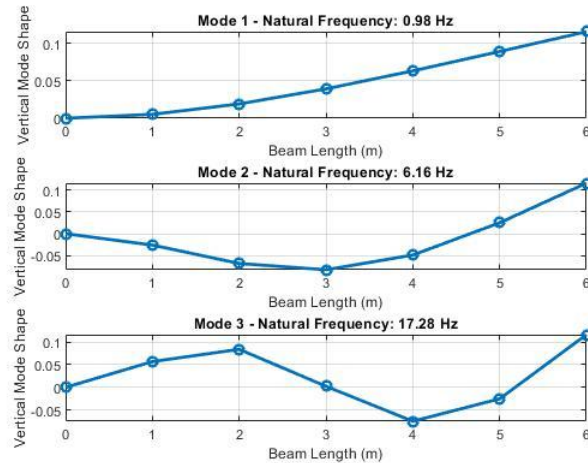


Figure 9: Mode Shapes Analysis

## DISCUSSION

Fig. 2, illustrates the deflection of the mid-span node when the applied load changes between 0 and 50 kN. The beam had a linear elastic behavior at the beginning until about 30 kN when it started to show some form of softening because of cracking of the concrete. Under the peak applied load of 50 kN, the deflection of the mid-span was about 5800 mm. This allows the early detection of load limits at which cracks occur to guide maintenance and reinforcement design.

Fig. 3, animation depicts how the beam is bending under load. Tension zones (bottom fiber in bending) are the initial places to see cracks. This behavior is predictive of the actual behavior of RC beam, concrete is weak in tension, and therefore cracks occurs first where bending moment is greatest. This is a progressive trend that proves the behavior of classical RC beam under increasing bending loads. High-risk factors can be identified to perform specific inspection and preventive reinforcement.

Fig. 4, shows initiation of cracks across steps of load. With progressive collapse behavior, cracks are observed to develop progressively as the load increases. The occurrence is similar to beam bending theory. The crack map may be used to drive inspection schedules since mid-span areas are risky areas. The development of steel stress is shown in

Fig. 5. The mid-span reinforcement reached a peak stress of approximately 5000 Mpa at full load which is lower than the yield stress of steel of 6000 Mpa. This means that concrete fractured, whereas the steel reinforcement was still in the elastic range. Well-structured reinforcement ensures ductility and thus, beams can experience massive deflections after cracking without leading to disastrous collapse.

Fig. 6, indicates the elastic strain energy and plastic /cracking energy versus load steps. It is noted that the plastic/cracking energy is nearly linear with load step, to a maximum value of about 50 kN•m and the elastic strain energy is almost negligible during the loading history. Establishes that the main energy absorbing mechanisms are cracking and plastic deformation. Failure modes can be predicted by energy based assessment.

Fig. 7, show beam deflection along the span and load step. The deflection is greatest at mid-span, which is in agreement with the theory of simple beams, and support regions are practically stationary. The spatial-temporal development of the cracks as indicated in crack surface fig. 8, offers a visual intuition of the progression of failure, Visualization helps to monitor the structure, and identifies areas most vulnerable to failure. Computed Natural frequencies of the 1st, 2nd, and 3rd bending modes can be seen in Fig. 9. Vertical deflection is mostly dominated by the first mode, and higher modes are

associated with local changes in curvature. Knowledge of mode shapes is important in evaluating dynamic responses. Vibration control or

damping mechanisms at the natural frequencies can be incorporated into the structural designs to increase resilience.

Table 2: Modes shape Natural frequencies

Mode	Frequency (Hz)	Interpretation
1	0.98	Fundamental bending at mid-span
2	6.16	Secondary bending near supports
3	17.28	Localized higher-order bending

#### IV. CONCLUSION

This paper developed a comprehensive numerical model to analyzed reinforced concrete beam under sudden support failure. Findings indicate that interaction between deflection, crack propagation, redistribution of stress, and energy dissipation governs structural behavior. The beam was elastic until approximately 30 kN when cracking began and gradually propagated resulting in higher deflection when mid-span was supported with the loss of mass. Although it cracked, the steel reinforcement remained within the elastic range. A significance increase in plastic energy (~50 kN m) indicates the role of energy dissipation to damage development, while modal analysis (60.98-17.28 Hz) help understand the dynamic characteristics. The integrated model captures the multi-response behavior of RC beams, and it is a comprehensive tool for predicting progressive collapse in case of support failure.

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