ISA Journal of Multidisciplinary (ISAJM)

Homepage: https://isapublisher.com/isajm/

Volume 3, Issue 2, March-April, 2026 ISSN: 3049-1851

The Power of Two: Integrating Conceptual and Procedural Methods in Solving Real-Life Algebraic Expression

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Received: 11.03.2026 | **Accepted:** 13.04.2026 | **Published:** 13.04.2026

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DOI: *10.5281/zenodo.19558803*

Abstract **Review Article**

The need for mathematical knowledge to be used in practical situations is increasingly being seen, and hence the need for both conceptual and procedural fluency in algebra is emphasized. Unfortunately, many students lack an interrelation between these two aspects of mathematical knowledge, making it difficult for them to use mathematics in practical situations. Therefore, the study sought to establish whether the IIA would help improve the students' understanding of conceptual and procedural skills to solve practical mathematical situations. A non-equivalent pretest-posttest quasi-experimental design was utilized using 37 students from two different schools in the Diffun I District. The statistical analyses conducted were frequency and percentage, mean, standard deviation, paired samples t-test, independent samples t-test, Levene's test, Shapiro-Wilk test, and Cohen's d. The pre-test showed the equivalence of the experimental and control groups. While in the post-test, the experimental group performed better than the control group with a mean percent score of 79.42 against 73.63 and a mean gain of 11.42 against 8.53, respectively. Considering that Cohen's d value is 0.447, the findings proved that the effect size is moderate, thus supporting the idea that there is an effective enhancement of students' problem-solving abilities by integrating conceptual and procedural knowledge. The study concluded that the use of the Integrated Instructional Approach is effective in improving learning outcomes in the field of algebra since it incorporates discussions based on concepts, procedural activities, and problem-solving exercises. It is advised for teachers to implement this instructional strategy to improve their students' problem-solving skills in algebra.

Keywords: Integrated Instructional Approach (IIA), Conceptual and Procedural Methods, Real-Life Algebraic Problem Solving

Introduction

Several studies have indicated over the years that for one to effectively engage in problem-solving in mathematics, especially algebra, one must develop conceptual understanding and procedural fluency skills. According to the literature available, the concept of understanding is the learning of mathematical concepts, ideas, and applications to help construct and apply knowledge in critical thinking and the practical use of mathematics (Childs, 2022; Hussein, 2022; Wedman & Bennet,

2025). Conceptual understanding involves the use of different representations, including symbolic, graphical, and algebraic representations, and encourages interaction with abstract mathematical concepts, such as rational numbers and algebra (Castillo et al., 2025; Bano & Singh, 2023). According to research literature, conceptual understanding enhances cognitive flexibility, creative problem-solving skills, and principled knowledge acquisition, thus helping generate procedures and retain mathematical knowledge (Elhilal, 2025; Bani Irshid et al., 2023; Andriatna et al., 2025).

However, procedural fluency is described as the capacity to execute mathematics procedures effectively, efficiently, and flexibly while relating them to concepts and adapting to new situations (NCTM, 2023; Bay-Williams & SanGiovanni, 2022; Ruiz et al., 2025). Procedural fluency instruction is direct, procedural fluency is practiced, procedural fluency is used wisely, and procedural fluency is judged thoughtfully (Blankman, 2023; CTL, 2023; Specjal, 2026). As defined in contemporary studies, procedural fluency cannot be considered rote memorization or rapid computation, but should be seen as an ability to analyze situations in terms of solving math problems (Almond, 2025; Hoskyns-Staples & Walters, 2025; Number Hive, 2024).

Further, according to recent research, conceptual understanding is closely linked with procedural fluency, and each of them contributes to the other during the acquisition of mathematics knowledge. While conceptual understanding assists in adapting procedures, procedural fluency helps to consolidate and enrich it, thus fostering critical thinking (Mitchell, 2025; Hakim & Yasmadi, 2021; Asilo-Ebisa & Lomibao, 2024; Childs, 2022). The current pedagogical approach stresses the significance of learning procedures where learners can observe and evaluate their ideas, make appropriate representations, and acquire procedural abilities effectively (Schumacher, 2019; Ji & Wong, 2025; Mathnasium of Lakewood CO, 2025). Research has proven that problem-based learning, hybrid model, and concept-before model not only improve procedural fluency skills but also support conceptual understanding (Learn Implement Share, n.d.; Colorado Department of Education, 2024; NCTM, 2023).

The empirical research evidence highlights the importance of combining procedural and conceptual knowledge. It has been proven that the two types of knowledge have reciprocal relationships since the higher the conceptual knowledge of learners, the better they can use procedures. On the other hand, carrying out procedures helps promote an understanding of concepts (Rittle-Johnson & Schneider, 2022; Braithwaite & Sprague, 2021; Barumbun & Kharisma, 2022). The use of conceptual knowledge in teaching algebra is known to promote improved understanding and the capacity to think critically, as well as the transfer of knowledge in various contexts (Ncube & Luneta, 2025). Incorporation of procedural and conceptual knowledge in teachers' professional development programs contributes to improved instructional practices in classrooms (Piñero Charlo & Canto López, 2026).

Some studies have established that certain interventions and instructional approaches can be effectively utilized to enhance students' understanding of concepts and procedures in solving algebra problems. Some of these include the utilization of GADS (Given, Asked, Drawings/Representations, Solution), TAPPS, storyboarding, and CRA approaches, which can assist learners to solve algebra problems by representing the problem, reflecting on, and visualizing its solution. Tiered interventions, repetitive procedural practice, and teacher professional development in solving algebra problems can also contribute to improving learners' algebraic problem-solving skills (Boger, 2025; Mohamoud, 2022; Chirinda, 2026; Hatisaru et al., 2022). In addition, research indicates that the use of non-routine problems, dynamic principles, and technology can promote learners' conceptual and procedural understanding in algebraic problem-solving situations.

Considering the above findings, this research attempts to apply the Integrated Instructional Approach wherein the use of conceptual knowledge

is consciously integrated with procedural knowledge in an attempt to enhance the problem-solving skills of Grade 6 pupils in solving authentic algebraic problems. Through concept-based, collaborative, and reflective learning, this study attempts to equip the learners with not only the required conceptual knowledge but also the procedures needed in problem-solving.

Methodology

A quasi-experimental research design, particularly the non-equivalent pretest-posttest control group design, was used to determine the effectiveness of the IIA in improving the conceptual understanding and procedural fluency of students in learning algebra. This type of research design is considered appropriate because it compares two naturally occurring groups without random assignment. It also measured the improvement in their performance before and after the experiment. In other words, the researcher was able to observe a cause-and-effect relationship within a realistic environment without the full extent of control found in an experimental study.

Grade 6 students enrolled in two public elementary schools in Diffun I District, namely Gulac Integrated School and San Isidro Integrated School, comprised the subjects of the study. Forty-two (42) students took part in the study, with twenty-three (23) belonging to the experimental group and nineteen (19) to the control group. Complete enumeration was applied since the total number of subjects was small. While the experimental group underwent the IIA, the control group received the traditional procedural approach. To ascertain the homogeneity of the variance, a test was performed before the experiment, which revealed that both groups had equal variability.

The data were obtained through the researcher's self-created algebra test instrument, which aimed to assess not only the conceptual knowledge but also the procedural skills involved in solving practical algebraic situations. The test instrument contained 30 questions given both pre-intervention and post-intervention. This test was created according to the MELCs and guided by the Table of Specifications to guarantee proper representation of topics and cognitive processes. The test was also subjected to expert validation by mathematics education specialists from the Department of Education and Quirino State University, with very high scores on content validity, clarity, organization, and acceptability.

There were three steps involved in collecting data for this study: pre-intervention, intervention, and post-intervention. For the pre-intervention phase, the experimental and control groups were tested using the pre-test to determine their existing levels of knowledge in algebra. This was done to ascertain that they were initially equivalent before being subjected to the treatment process. For the intervention phase, the experimental group was exposed to the Integrated Instructional Approach by combining procedural and conceptual learning in real-world problem-solving, scaffolding, collaboration, and reflection. On the other hand, the control group underwent instruction using the procedural approach in solving problems one after another. The intervention was conducted for two weeks during the scheduled class time. Finally, in the post-intervention phase, the experimental and control groups were tested using the post-test to identify their level of improvement in performance.

The data were statistically analyzed to establish whether the program intervention is effective or not. The use of descriptive analysis involved frequency, percentage, mean, and standard deviation to describe the profiles of the respondents and their performance. On the other hand, inferential analysis used paired sample t-test and independent samples t-test to establish any significant difference among or within respondents. Levene's Test was used to evaluate the homogeneity of variance, while the Shapiro-Wilk Test was used to establish the normality of data distribution. Additionally, the effect size of the intervention was determined using Cohen's d.

Results and Discussions

Table 1. *Frequency and Percentage Distribution of Participants by Profile across Experimental and Traditional Groups*

Profile	Particulars	Group			
		Experimental Group	Percent	Traditional Group	Percent
Sex	Male	8	40.00	6	35.29
	Female	12	60.00	11	64.71
Age Mean = 11.14	11 years old	16	80.00	16	94.12
	12 years old	4	20.00	1	5.88

Table 1 shows the frequency and percentage distribution of the participants' profiles across the experimental and traditional groups. It is important to note that the number of participants was slightly reduced from 23 and 19 to 20 and 17 in the experimental and traditional groups, respectively, due to absenteeism during some of the instructional and assessment sessions. From the table, one can observe that in the experimental group, there were 8 males (40 percent) and 12 females (60 percent), while in the traditional group, there were 6 males (35.29 percent) and 11 females (64.71 percent). Regarding age, the experimental group had 16 students who were 11 years old (80 percent) and 4 students who were 12 years old (20 percent). In the traditional group, 16 students were 11 years old (94.12 percent), and 1 student was 12 years old (5.88 percent).

The majority of the respondents were 11 years old and female for both groups. This shows a similarity in composition in terms of age and sex between the experimental and traditional groups. The average age of both groups is the same as the average age of a Grade 6 student, which means demographic factors such as age and sex do not affect the difference in learning outcomes.

Table 2. *Pretest Mean Percent Score in Solving Real-Life Algebraic Problems of the Participants*

Group	N	Mean Percent Score	SD	t	p-value	Decision
Experimental Group	20	68.00	4.79	2.015	0.052	Fail to reject Ho
Traditional Group	17	65.10	3.79			

p-value ≤ 0.05 *is significant*

Table 2 shows a comparison of the pretest mean percent score of both the experimental and traditional groups on solving real-life algebraic problems, where the experimental group obtained a score of 68.00 percent and the traditional group obtained a score of 65.10 percent. Moreover, the p-

value of 0.052 revealed that there was no significant difference between the two groups, implying that both groups were at the same level of proficiency. This indicates a fair level of performance in measuring the effects of the instructional intervention. In addition, the literature showed that the performance of learners in solving problems at the start of the instructional session is often dependent upon learners' prior knowledge and exposure to problem-solving rather than the method of instruction (Kablan & Süzer Uğur, 2021; Shawan et al., 2021; Wahyun et al., 2024; Or & Bal, 2023; Legarde, 2022). On the other hand, the similarity in performance during the pretest can also be further substantiated by Mohamoud (2022), and it was established that the students initially had difficulties in procedural fluency and conceptual understanding in algebra, but they showed a marked improvement in performance after practice. Moreover, it was found by Ali (2023) that there were no significant differences in the ability of the students in solving algebra word problems, thus further substantiating the fact that struggling in the beginning might be attributed to the understanding of the context of the problem and not necessarily to differences in groups. This can be substantiated by the findings of Pramayudi et al. (2020) and Md Nasir et al. (2021).

Generally, the similar results of the pretests justify the appropriateness of the application of the Integrated Instructional Approach (IIA) since the approach integrates the conceptual and procedural knowledge. Various research studies have confirmed that the combination of conceptual and procedural knowledge enhances adaptive and strategic problem-solving skills (Pelayo et al., 2023; Gavaz et al., 2021; Mohamoud, 2022).

The results are consistent with the homogeneity test as reflected in Table 1, which also showed that the two groups were comparable in terms of prior knowledge and variability in solving real-life algebraic problems.

Table 3. *Comparison of Pretest and Post-test Mean Percent Score in Solving Real-Life Algebraic Problems of the Participants in the Experimental Group*

Group	N	MPS	SD	t	p-value	Decision
Pretest	20	68.00	4.79	-8.755	< .001	Reject Ho
Posttest	20	79.42	3.39			

Table 3 shows the result of the comparison of the pretest and posttest mean percent scores of the experimental group on solving real-life algebraic problems. From the result above, it is evident that there is an improvement in the students' performance from the pretest (68.00 percent) to the posttest (79.42 percent). In addition, it is evident that there is a highly significant difference between students' performance in the pretest and posttest, as indicated by $p < 0.001$. This improvement in students' performance on real-life algebraic problems indicates that the use of an integrated instructional approach was effective in improving students' performance on real-life algebraic problems. This improvement supports the literature on mathematics instruction, which indicates that students improve their conceptual and procedural knowledge of mathematics through structured and repetitive exposure to problem tasks under a structured and guided instructional framework (Pelayo et al., 2023; Wahyun et al., 2024). Moreover, students' improvement on real-life algebraic problems also indicates the significance of strategy instruction and reflective thinking in mathematics instruction (Or & Bal, 2023; Pramayudi et al., 2020).

The findings of Mohamoud (2022) support this pattern, showing that ninth-grade students initially struggled with procedural fluency and abstract algebraic concepts, but daily structured fluency practice for five weeks significantly improved their procedural problem-solving efficiency. Similarly, AlMutawah et al. (2019) observed that students with good conceptual understanding could use procedural knowledge better in real-life problem contexts, and instruction improved posttest performance. Khansila et al. (2022), while focusing on geometry, demonstrated that integrating conceptual understanding and procedural skills resulted in significantly higher posttest scores, providing a parallel to algebraic problem-solving interventions. Furthermore, the Concrete–Representational–Abstract (CRA) approach used by Bernadez and Montero (2025) showed that emphasizing conceptual understanding before procedural fluency produced substantially higher posttest scores, particularly in applied, real-life algebra tasks.

Collectively, the aforementioned studies provide credence to the current findings by supporting the idea that an intervention consisting of the integration of conceptual and procedural instruction would prove effective in improving the performance of the students in solving complex and real-world applications of mathematics. The improvement noted among the experimental group's posttest scores is not only an indication of the improvement of procedural accuracy but also of the improvement of strategic thinking skills, thereby supporting the intervention approach adopted in the present study.

Table 4. *Posttest Mean Percent Score in Solving Real-Life Algebraic Problems of the Participants*

Group	N	Mean Percent Score	SD	t	p-value	Decision
Experimental Group	20	79.42	3.39	4.029	< .001	Reject Ho
Traditional Group	17	73.63	5.28			

p-value ≤ 0.05 *is significant*

Table 6 shows the posttest mean percent scores of both the experimental and traditional groups in solving real-life algebraic word problems. The results revealed that the experimental group with a mean percent score of 79.62 performed better than the traditional group with a mean percent score of 73.63 in solving real-life algebraic problems. The results were found to be statistically significant at $p < 0.001$. These results are in line with the literature, which stresses that it is more effective in improving students' problem-solving skills if an approach used combines conceptual and procedural knowledge (Pelayo et al., 2023; Wahyun et al., 2024; Mohamoud, 2022). Moreover, it is evident from the results of the experimental group that students need to be exposed to real-life and non-routine problems, as it has been found to improve students' flexibility and adaptive strategy use (Gavaz et al., 2021; Evans et al., 2021; Or & Bal, 2023; Legarde, 2022).

This was further evidenced in a study done by Mohamoud (2022), where it was found that the ninth-grade Algebra 1 students had difficulties in procedural skills and conceptual understanding in solving algebraic problems. However, after they were given a five-week daily fluency skill-building program in procedural skills and conceptual understanding in number sense, integer operations, and solving linear equations and inequalities, it was found that the students showed statistically significant gains in procedural fluency. In addition to

that, it was found that the students showed improvements in procedural fluency in solving algebraic problems, as observed in the experimental group in the current study.

Furthermore, these findings also validate previous studies that emphasize traditional approaches that focus on procedural aspects without linking them to conceptual understanding are less effective in promoting the development of higher-order thinking skills (Shawan et al., 2021; Pramayudi et al., 2020).

Table 5. *Comparison of the Mean Difference of the Posttest and Pretest in Solving Real-Life Algebraic Problems of the Participants*

Group	Mean Gain	SD	t	p-value	Decision
Experimental Group	11.42	5.83	1.354	0.184	Fail to reject Ho
Traditional Group	8.53	7.14			

p-value ≤ 0.05 *is significant*

The comparison of mean gains between the experimental and traditional groups in addressing real-life algebra problems is illustrated in Table 7. From the analysis of the findings presented, it is evident that the experimental group (11.42) showed a higher mean gain compared to the traditional group (8.53). This indicates that the participants exposed to the Integrated Instructional Approach showed greater enhancement in their performance than those who were taught using the traditional method. Nonetheless, this observed difference is statistically insignificant ($p = 0.184$). Such findings resonate with prior research works, which have highlighted that even though integrated approaches may record higher mean gains than conventional approaches, they may not be statistically significant particularly when the performances of both groups were initially equal (Wahyun et al., 2024; Pelayo et al., 2023).

The absence of a significant difference might also be attributed to the characteristics of the individual learners. As Kablan & Süzer Uğur (2021) and Shawan et al. (2021) have noted, cognitive aspects of prior knowledge and learning styles are related to teaching methods in determining gains in performance. Therefore, a slight increase in an integrated method of teaching may be regarded as a significant effect on the results of learning, although more time or trials may be required to attain a significant effect.

Table 6. *Effect of Integrated Instructional Approach in Enhancing the Performance of the Participants*

Group	Mean Gain	Mean Difference	*t*	p-value	*Cohen's d*
Experimental Group	11.42	2.89	1.354	0.184	0.447
Traditional Group	8.53				

p-value ≤ 0.05 *is significant*

Cohen's d: small effect size = around 0.2; medium effect size = around 0.5; large effect size = around 0.8 or higher

From table 6, it can be noted that it reflects the effect of the integrated instructional approach on participants' performances in real-life algebraic problems. The experiment's findings reveal that there was an improvement in the experimental group since it had a higher mean gain of 11.42 in comparison to the traditional group that had a mean gain of 8.53. Although the difference was not statistically significant at 0.184, a medium effect size was noted at 0.447, indicating improved performance of students through the application of the integrated instructional approach.

The Cohen's d value of 0.447 provides a more meaningful insight into the practical effect of the intervention. The value falls within the range of a moderate effect size, indicating that the Integrated Instructional Approach (IIA) has a noticeable and educationally meaningful impact on students' performance on algebraic problems in practical terms. This implies that, although the difference was not strong enough to be statistically significant – possibly due to relatively small sample size – the intervention still produced a moderate improvement in learning outcomes.

This is in line with the existing literature on teaching methods that encourage scaffolding for both conceptual and procedural knowledge, which has shown positive effects on the improvement of problem-solving skills, although statistical significance is not immediately achieved (Pelayo et al., 2023; Wahyun et al., 2024; Mohamoud, 2022). For instance, Mohamoud (2022) noted that ninth-grade Algebra 1 students showed significant improvement in procedural fluency and efficiency in problem-solving after being that college students exposed to the dynamic exposed to a daily fluency skill-building intervention, in which a medium effect size was shown. Another study by Peresuode and Patience (2024) noted principles of Zoltan Dienes showed significant improvement in post-test procedural and conceptual knowledge of algebra. This indicates that the inclusion of manipulatives and scaffolding in teaching is significant in showing positive effects on performance outcomes, as shown in the effect size.

Further support comes from Gavaz et al. (2021) and Evans et al. (2021), who emphasize that scaffolded guidance, strategy instruction, and reflective tasks contribute to improvements in reasoning, strategic flexibility, and procedural accuracy. On one hand, repeated exposure to non-routine and real-life tasks helps in the development of adaptive reasoning skills, as emphasized by the works of Or & Bal (2023) and Legarde (2022), which showed the significance of the results despite the non-significant differences as indicated by the statistical tests.

On the other hand, the cognitive characteristics of the students, i.e., the prior knowledge of the students and the learning styles of the students, are reported to influence the performance of the students (Kablan & Süzer Uğur, 2021; Sita Pramayudi et al., 2020). Hence, the medium effect size found in the present study supports the idea that the integrated approach would positively impact students' performance on real-life algebra problems, as suggested by recent studies.

Based on the outcomes of the experiment, one can conclude that integrating the conceptual and procedural modes of teaching Algebra is an effective way to enhance students' mathematical literacy. In addition, the effectiveness of the approach implies that the traditional ways of teaching Algebra should be changed, and educators should adopt more progressive methods of instruction that would enable the students to have a deeper understanding of the subject matter, apply what they learn in practice, and be actively involved in the educational process. School administrators and developers of curricula should pay attention to the issue of creating the necessary learning materials and training programs aimed at improving the ways of teaching mathematics, as a rule. Although the difference between the means gained by both groups was insignificant, a positive trend and medium effect were observed.

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The key concern of this study was to explore whether an Integrated Instructional Approach would help improve Grade 6 learners' performance in dealing with real-life algebraic problems. In the process, there are strategies that enhance the learning experience through both conceptual understanding and procedures, but they are not part of the main variables investigated in the research. In terms of scope, the study only focused on algebra problems in real-life situations. It was carried out in purposively selected schools from a particular geographical location, and therefore, the findings cannot be generalized to other contexts outside of the sample population. Besides, it focused on the short-run results without evaluating any long-term benefits that may accrue to the subjects in the future. Performance was gauged based on well-structured assessment instruments representing real-life situations, but the study excluded other confounding factors that may affect students' performances, such as socio-economic background, among others.

ACKNOWLEDGMENT

The researcher expresses his sincere gratitude to his adviser, panel members, and institution for their guidance and support. He also extends his appreciation to the respondents for their valuable participation. Finally, he thanks his family for their encouragement and the Almighty for the strength and wisdom to complete this study.

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