



# Integration of Comparison–Addition Model (CAM) and Relaxation Principle (RP) Framework for Efficient Route Tracking in a Dense Network Graphs

Uchenna E. Mba<sup>1\*</sup>, Njideka N. Mbeledogu<sup>2</sup>, Chioma V. Anikwe-Onyiwa<sup>3</sup> Chukwuemeka Okpara<sup>4</sup>

<sup>1,3,4</sup> Computer Science Department, David Umahi Federal University of Health Sciences, Uburu, Ebonyi State

<sup>2</sup> Computer Science Department, Nnamdi Azikiwe University, Awka, Anambra State

Received: 11.05.2026 | Accepted: 09.06.2026 | Published: 14.06.2026

\*Corresponding Email: [mbaue@dufuhs.edu.ng](mailto:mbaue@dufuhs.edu.ng)

DOI: [10.5281/zenodo.20686165](https://doi.org/10.5281/zenodo.20686165)

## Abstract

## Original Research Article

Distance calculation has incredibly evolved to be a pivotal anchor and a huge relief for commuters, data transmission settings, packet delivery and those seeking optimal paths to save time and resources. The main concern has driven from latency, resource consumption to optimized completeness in real-time bearing in mind edges and vertices configuration; whether negative or non-negative. When applied in air routing, it increases efficiency and productivity in the aviation sector and as well, improves the overall performance. When applied in data routing network - frames and packet transmission are relayed in wholeness with packet set priorities. When applied in unclassified network graph, there is need to increase the turnaround time for node-to-node traversals with mixed edge and vertices. This research paper is focused on integrating the two foremost traditional shortest path algorithms: Modified Dijkstra and Bellman-Ford. The former is fundamental in finding an optimal route in a non-negative network using complex and multiple parameters led by Comparison-Addition Model (CAM) while the later is a robust and highly efficient shortest path algorithm with full optimization in negative graphs led by Relaxation Principle (RP). The implementability and applicability of the algorithms, and its approaches for shortest path computation bearing mind its unique characteristics, configuration and structures. Shortest path calculations from single source to multiple destinations in a digraph network involve a designated route through the edges and vertices which are meant to be linked to the entire nodes in the network. CAM and RP are methodologies used to further analyze and compare the algorithms and routing patterns called traversals. From the source to destination nodes. The process ensured reduction in the number of iterations; quick and optimal route discovery were enhanced, and the entire network model experienced a minimized latency power. Using CAM and RP in network routing will in no doubt improve data route movement, transmission and packet delivery regardless of the network graph configurations, structures and density.

**Keywords:** Comparison-Addition Model (CAM), Shortest Path Algorithm, Optimization, Traversal, Routing, Congestion Reduction, Flight Path, Dijkstra's Algorithm, Bellman-Ford Algorithm.

Copyright © 2026 The Author(s). This is an open-access article distributed under the terms of the Creative Commons Attribution-NonCommercial 4.0 International License (CC BY-NC 4.0).



## 1.1 Introduction

Shortest path problem solving has long rested on fundamental algorithms with origins in graph theory, such as Bellman-Ford and Dijkstra's. Despite their successes, the growing complexity and dynamic nature of contemporary networks have exposed their shortcomings. Advanced approaches include heuristic, hybrid and artificial intelligent (AI) driven methods have been developed to get around these challenges. This innovation has improved computing efficiency, security and adaptability (Hadi & Ibrahim, 2025). Shortest path algorithms are built on defined foundations in graph theory, that depict networks as graphs composed of nodes (representing devices) and edges (representing connections). Basic algorithms such as Bellman-Ford and Dijkstra's were the first to tackle the single-source shortest path problem. Due to their efficiency and ease of use, these conventional techniques are still widely used today and have formed the basis of modern routing protocols. Bellman Ford, for instance, has proven to be robust in situations when edge weights are negative, and Dijkstra's technique is crucial for link-state routing protocols. With the increasing sophistication and breadth of networks, traditional shortest path approaches have faced challenges in handling resource constraints, large datasets, and shifting topologies. To address these problems, emphasis has been laid on developing complex algorithms that incorporate heuristics, hybrid approaches and artificial intelligence. Some optimization modules like the ant colony takes advantage of natural foraging behavior to determine the optimal routes and block chain-based solutions enhance routing security by providing transparent and unchangeable path decisions. With these advancements, algorithms may now adapt dynamically to changing network conditions and increase computational efficiency. However, they have been limits such as inflexibility needed in achieving the shortest path in rapidly changing network topologies. Based on this, a need for optimal and adaptable approach is desired (Chadjou & Kyamakya, 2015; Hadi & Ibrahim, 2025).

A densely populated network graph is characterized by a large number of interconnected vertices and

edges, present significant challenges in route tracking due to increased computational complexity, path redundancy, and scalability concerns. Using traditional shortest-path algorithms such as Dijkstra's Algorithm and Bellman-Ford's Algorithm may rely primarily on edge relaxation techniques to determine optimal routes. However, in highly connected networks, these approaches may experience increased processing overhead as the number of possible paths grows exponentially. To integrated route-tracking framework that combines the Comparison-Addition Model (CAM) with the Relaxation Principle (RP) to improve path discovery efficiency and route optimization within dense network environments in not just and improvement, but time saving venture (Mba et al, 2026).

The Comparison-Addition Model which comes from the modified Dijkstra's Algorithm introduces a systematic mechanism for comparing alternative path costs while incrementally accumulating edge weights throughout network traversal. Concurrently, the Relaxation Principle continuously updates tentative shortest distances whenever a more optimal route is discovered regardless of its edge configurations. The integration of CAM and RP enables the algorithm to evaluate competing routes through comparative cost analysis while dynamically refining path estimates using relaxation operations and negative edges accommodation. This dual mechanism reduces unnecessary path explorations, enhances convergence speed and improves route selection accuracy. The integrated CAM-RP framework is particularly suitable for applications involving transportation networks, communication systems, airline route optimization, intelligent traffic management and large-scale data packet routing for di and undirected graphs. By combining additive path accumulation with continuous relaxation-based optimization, the model achieves improved route-tracking performance, reduced computational redundancy and enhanced scalability in dense graph structures. The framework provides a robust foundation for developing next-generation shortest-path and route-optimization algorithms capable of handling increasingly complex network topologies while maintaining high levels of accuracy and

computational efficiency irrespective of next-edge and next-vertex definitions (Mba et al, 2026, Hadi & Ibrahim, 2025, Mba & Mbeledogu, 2024).

Rodrigue (2024) suggested that the core advantages of air transportation are speed and flexibility in network configuration. Air routes are practically unlimited but denser over the North Atlantic, inside North America, Europe, and over the North Pacific. More recently, air transportation has accommodated growing quantities of high-value freight and is playing an increasing role in global logistics. In Nigeria, air transportation generates some financial benefits due to the economic activities. Nigeria operates a certified Category 1 aviation. This is because of its high performance in the air transport sector. Based on this certification, Nigeria's air safety status has improved and now presents the country in the premier League of Nations that are highly rated in air transport. The growth of the ability and the need to transport large quantity of goods or number of people over long distances at high speeds in comfort and safety have been the index of technological progress in air transportation system. The major components of this system include airports, air traffic control system, aircraft and airlines. Any change in each of these components will have important consequences for the future of aviation industry (Nissalike, 2025). As one would expect any infrastructure that has such ubiquitous reach, air transportation systems face significant challenges in Nigeria. (Saheed et al., 2015) identified some key difficulties as high cost of operation, poor maintenance of airports and aircrafts, and insufficient financial resources.

Mba and Mbeledogu (2024) adopted the Modified Dijkstra's Shortest Path Algorithm (MDSPA) to investigate the shortest path in Terminal Maneuvering Area (TMA) using Comparison Addition Model (CAM) flexible with permutation and combination capabilities. TMA addressed the

problem of distance and altitude miscalculations where enroute airspace begins in air transportation system for digraph. The efficiency and reliability of transportation and data routing across networks depends heavily on the means and routing algorithms. Aside the tremendous benefits associated with air transportation, considering it as an overlay in a routable hybrid model of Modified Dijkstra's and Bellman Ford algorithms, knowing their uniqueness makes the entire research palpable. Shortest path techniques are necessary to reach optimal performance and engaging algorithms to determine the optimal data transmission channels aimed to reducing critical characteristics such as latency, attenuation, cost, and energy consumption while maintaining network reliability and stability over a dedicated route is a clear definition of optimality.

### **Routing Table/Information Base (RIB)**

To effectively build a framework for the integration of CM-RP model, a routing table or routing information base (RIB) which defines a data table stored in a routing network that host that lists the routes to particular network destinations and in some cases, metrics (distances) associated with those routes must be formulated. The routing table contains information about the topology of the network immediately around it and can be easily visited during route traversals and optimization within the network model graph (Kinza, n.d).

### **Shortest Path Algorithm Classification**

Shortest path algorithm can be streamlined to two primary and distinct groups. They are classical and heuristic. (Mba & Mbeledogu, 2024; Hadi & Ibrahim, 2025). Figure 1.3 gives a pictorial illustration of the classification holds more details.

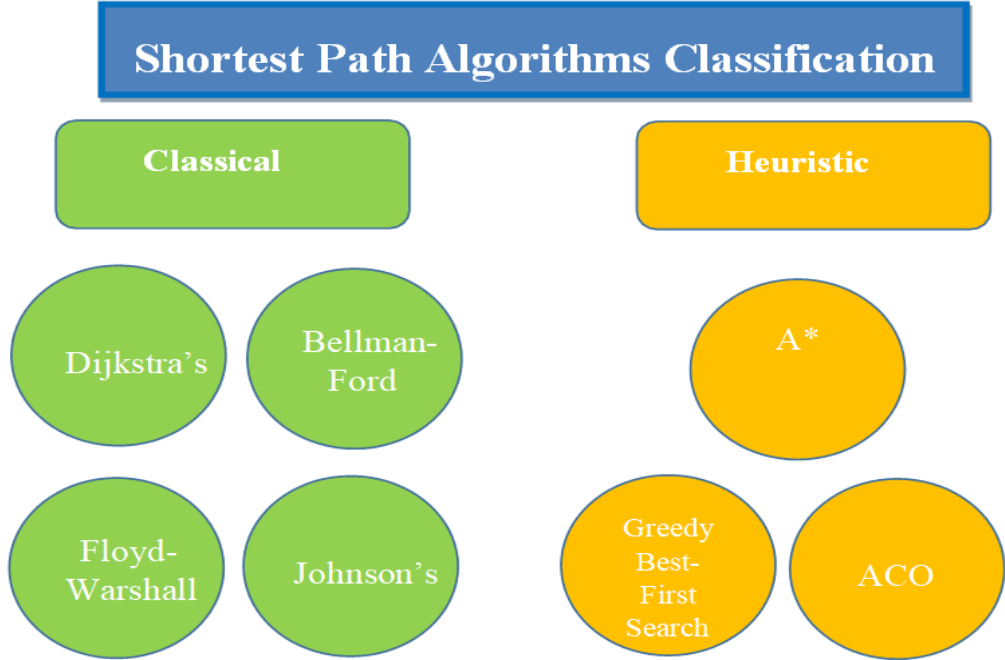


Figure 1.1: Shortest Path Algorithms Classification (Hadi & Ibrahim, 2025)

**Classical Algorithms for Shortest Path**

Deterministic techniques known as classical algorithms ensure the best answers to shortest path issues. Examples include Bellman-Ford, which can handle distributed computations with negative weights, and Dijkstra's, which is appropriate for graphs with non-negative weights. They serve as the cornerstone of reliable and effective network routing, and are independently called single source shortest path (SSSP) problem algorithm (Mba & Mbeledogu, 2024).

**A. Dijkstra's Algorithm**

The Dijkstra's shortest path algorithm is the most commonly used to solve the single source shortest path problem today. This algorithm avoids the use of negative cycles by assuming that all the edges are positive. This allows the algorithm to use a heap where the key of a vertex is the length. Beginning at the source vertex, each

adjacent vertex is assigned the cost value of edge joining them. Next, each vertex will process the least accumulated cost and assign the accumulated cost plus the edge cost to each of its adjacent vertices. This step is repeated until each vertex has been processed. If a vertex is revisited, the algorithm will assign the new cost if it is lower than the currently assigned cost (Mba & Mbeledogu, 2024; Hadi & Ibrahim, 2025).

**B. Bellman-Ford Algorithm**

This is a graph search technique that finds the shortest path between a specific source vertex and each other vertex in the graph. This method can be applied to both weighted and unweighted graphs. Similar to Dijkstra's shortest path algorithm, the Bellman-Ford method is guaranteed to find the shortest path in a graph. Bellman-Ford is more adaptable than Dijkstra's method since it can handle graphs with negative edge weights, even if it is slower. It is crucial to

keep in mind that in a graph with a negative cycle, there is not a shortest path. If the road continued to circle the negative cycle indefinitely, the cost would decrease even if the journey duration increased. Bellman Ford thus has the added advantage of being able to recognize negative cycles.

### C. Floyd-Warshall Algorithm

This is one method for figuring out the shortest paths between each pair of nodes in a network. It uses a dynamic programming technique to determine the shortest paths for the entire graph, progressively coming up with solutions to smaller sub-problems. The method is applicable to both directed and undirected graphs, and is particularly effective for dense graphs. However, the graph must not have negative weight cycles because this would result in undefined shortest paths. The process begins by initializing a distance matrix, where each entry represents the shortest distance between two nodes.

### D. Johnson's Algorithm

It emphasizes on a technique for figuring out the shortest paths between each pair of nodes in a weighted graph. Because it combines the benefits of Bellman-Ford's and Dijkstra's algorithms, it works particularly well with sparse graphs. The unique feature of Johnson's Algorithm is that it can handle graphs with negative edge weights as long as there are no negative weight cycles. The algorithm first reweights the edges of the graph to eliminate negative weights. The Bellman-Ford algorithm is used to determine the potential value of each node, and then all of the graph's edge weights are adjusted. This reweighting ensures that all edge weights become non-negative while preserving the relative order of shortest pathways. The approach uses Dijkstra's algorithm to determine the shortest pathways from each node after reweighting. Since Dijkstra's algorithm works well for networks with non-negative weights, this technique allows Johnson's Algorithm to perform better for sparse graphs than other all-pairs shortest path techniques.

### E. A\* Search Algorithm

A popular heuristic-based approach for determining the shortest path between a source node and a target node in a graph is the A\* algorithm. It works especially well in applications with wide search spaces, such as game development, robotics, and navigation systems. The A\* algorithm balances computational efficiency and optimality by combining the advantages of Greedy Best-First Search and Dijkstra's Algorithm.

A\* achieves its performance by using a cost function to guide its search. The cost function is defined as:

$$f(n) = g(n) + h(n)$$

Where:

- $g(n)$  is the actual cost from the start node to the current node  $n$ .
- $h(n)$  is the heuristic estimate of the cost from  $n$  to the target node.

The heuristic  $h(n)$  is a crucial component that establishes the algorithm's efficiency. It must be acceptable (never overstate the genuine cost) in order to guarantee optimal solutions. The method iteratively investigates nodes with the lowest  $f(n)$  value to ensure that the routes most likely to lead to the target are examined first. If the heuristic is well-designed, A\* can significantly reduce the search space when compared to other shortest path algorithms. Because it enables the heuristic to be tailored for specific applications, A\*'s versatility is highly valued by many (Hadi & Ibrahim, 2025).

### F. Greedy Best-First Search algorithm

Greedy Best-First Search is a heuristic-based path finding method that looks into nodes that seem to be closest to the objective based on a heuristic assessment. "Greedy" refers to its method of continuously choosing the node with the lowest heuristic value in an attempt to reach the goal as quickly as feasible. Unlike other algorithms, such as A\* or Dijkstra's, which consider both the expected cost to the objective

and the actual cost of accessing a node, Greedy Best-First Search alone employs the heuristic function to guide its decisions. The algorithm evaluates its neighbors based on their heuristic values, starting at the source node. After selecting the neighbor that appears to be closest to the goal, it moves to that node. During this process, the algorithm iteratively grows the node with the smallest estimated distance to the destination. Because of its simple, goal-oriented approach, the algorithm can often find a path to the objective quickly, especially in simple or well-structured graphs. However, because greedy best-first search disregards the actual cost of reaching a node, it does not yield the shortest path. In other cases, the heuristic function may even select a longer, less optimal path if it produces estimates that are not correct.

### G. Ant Colony Optimization (ACO) algorithm

Ant Colony Optimization (ACO) is a technique that was inspired by the way ants forage for food in the wild. In the wild, ants initially roam around aimlessly, but when they return to the colony after locating food, they leave behind pheromone trails. Other ants, which are more likely to follow paths with higher pheromone concentrations, pick up these tracks. Eventually, more ants prefer the shortest road since it gathers the most pheromone from frequent use. ACO computationally simulates this behavior to address complex optimization problems, especially those involving paths, such as the traveling salesman problem or network routing. The algorithm initially visualizes the problem as a graph, where nodes represent decision points (e.g., cities on a route) and edges reflect relationships with associated costs (e.g., distances). The graph is traversed by artificial "ants" that construct solutions. Each ant makes probabilistic decisions on which path to follow next based on two factors: problem-specific heuristic information, such as the distance to the next node, and the quantity of pheromone on each edge, which reflects the cumulative desirability of that path. As the ants complete

their journeys, the algorithm evaluates the quality of their solutions. The pheromone on less appealing paths is allowed to progressively fade away, while more pheromone is introduced to the edges of paths that lead to better solutions. This evaporation prevents the algorithm from becoming stuck in less-than-ideal solutions by reducing the influence of suboptimal paths. All things considered, Ant Colony Optimization is an intriguing illustration of how strong computational methods can be inspired by natural systems. It is a powerful and adaptable tool for resolving optimization issues in a variety of fields since it can replicate the decentralized and self-organizing behavior of actual ants.

### 1.3 Intelligent Hybrid Model

The foundational mechanism of an intelligent hybrid model revolves around its Comparison-Addition Model (CAM), which serves as the primary decision-making framework for evaluating and promoting candidate paths in a network. CAM operates by systematically assessing the temporary labels assigned to each node during the traversal of the network graph. Each node in the network maintains a temporary label (TL) representing the cumulative cost or weight of reaching that node from the source and a permanent label (PL) indicating the finalized minimum cost once the node is selected as part of the shortest path. The CAM mechanism compares all temporary labels in a given iteration, identifies the node with the lowest cumulative weight, and promotes it to a permanent label, ensuring that the route selection is both correct and generally optimal within the graph. This procedure prevents premature selection of suboptimal paths, which is a common limitation in conventional Dijkstra's or similar algorithms when handling complex network topologies with multiple alternative routes. Complementing CAM, the reversed Comparison-Addition Model (rCAM) applies a similar logic but in reverse order, traversing the network from destination nodes toward the source. This backward evaluation ensures that any feasible routes that may have been overlooked in the forward traversal are captured during the return trip, providing

bidirectional robustness and significantly improving the reliability of the path finding process. In addition to these label-based models, the system integrates permutation and combination techniques to exhaustively generate all possible sequences of intermediate nodes between the source and destination. By calculating the permutations of node sequences and combining them systematically, the model is able to produce a complete set of candidate routes rather than limiting itself to a single deterministic path. This approach is particularly important in real-world networks where multiple near-optimal alternatives may exist due to variable traffic conditions, operational constraints, or edge weight fluctuations. To organize and evaluate the traversal paths, the model employs a Report Evaluation Matrix (REM), which serves as a structured framework to classify, rank, and score each calculated route against a series of evaluation metrics, including cumulative cost, expected travel time, network congestion impact, and operational feasibility. REM effectively allows the system to quantify the performance of each path and select those that meet both efficiency and practical operational criteria. By integrating CAM, rCAM, permutation/combination generation, and REM, the model achieves a comprehensive mechanism that balances exhaustive path exploration with computational efficiency, ensuring that all feasible, alternative, and optimal paths are identified and ranked systematically during final computation, regardless of whether edge weights in the network are negative or non-negative, a capability that extends the applicability of the model to complex and heterogeneous graph scenarios.

In addition to its core path-evaluation mechanisms, the IHM incorporates a suite of performance optimization parameters designed to enhance operational efficiency and scalability across complex networks. Time Factor Optimization (TFO) is central to the model's temporal efficiency, as it allows the system to prioritize routing paths that minimize overall travel or traversal time. TFO evaluates not only the static distance or cost associated with network edges but also dynamic factors such as expected delays, traversal speeds, and time-sensitive constraints relevant to operational environments like

aviation networks. By embedding TFO, the system ensures that the selected paths are temporally optimal and realistically feasible, reducing latency and improving responsiveness in time constrained and critical applications. The Congestion Reduction (CR) parameter addresses network flow and load balancing by analyzing traffic density, route utilization, and potential bottlenecks. CR evaluates route paths for their propensity to alleviate or exacerbate congestion, promoting alternative routes where necessary to distribute network load evenly. This reduces the risk of bottlenecks, prevents overuse of specific nodes or edges, and enhances the overall stability of the network under high-demand or densely populated network graph. Memory Utilization (MU) engages the combinatorial complexity introduced by permutation-based route generation. MU ensures that the system efficiently manages computational resources, including data structures that store the route paths, labels and evaluation matrices, minimizing memory overhead while maintaining performance. This is particularly important when working with large-scale or densely connected networks, where inefficient memory usage can lead to excessive computational latency or system instability. The Reversibility Pattern (RP) introduces bidirectional adaptability into the model. RP allows the system to dynamically verify that route paths remain optimal and feasible when evaluated in both forward (source to destination) and reverse (destination to source) directions. This is particularly valuable in networks subject to changing conditions, where a previously optimal path may become suboptimal due to real-time variations in edge weights, traffic flow or operational constraints.

The Intelligent Hybrid Model (IHM) further distinguishes itself through the integration of artificial intelligence (AI) through Random Forest Learning, which significantly enhances the system's adaptability and predictive capabilities. By leveraging random forest algorithms, the model can classify and learn from large datasets of network behavior, capturing patterns, correlations, and anomalies that would be difficult to encode manually through static rules or heuristics. In particular, the model utilizes arbitrarily generated datasets sourced from the Federal Aviation Authority of Nigeria

(FAAN), simulating real-world operational scenarios such as variable air traffic conditions, runway usage, flight scheduling conflicts, and route congestion. These datasets allow the AI component to train on diverse and realistic network conditions, enabling it to refine route classification, identify trends, and predict optimal paths under changing circumstances. The AI layer works synergistically with the CAM and rCAM mechanisms, using the insights from learned patterns to adjust temporary labels, prioritize route paths in the REM and even recommend alternative routes that human operators may not have considered. This adaptive learning capability ensures that the system can improve over time, dynamically responding to new network conditions, unforeseen events or irregular edge weight distributions, including negative or non-negative values. Additionally, the integration of AI allows the model to perform sophisticated edge weight scenario analysis, evaluating the impact of potential

operational changes before they occur, thereby supporting decision-making in complex environments like aviation sector. By combining classical algorithmic rigidity with AI-driven adaptability, the model achieves a level of intelligence and operational foresight that positions it as a highly advanced solution for modern network routing challenges, capable of delivering optimal and resilient paths in large and densely populated network model graph (Mba et al, 2026).

### Intelligent Hybrid Network Modeling

The network model for an air route and flight dispatch system was abstracted as a graph (see Figure 1.2). The algorithm works efficiently for both non-negative directed and undirected graph network systems.

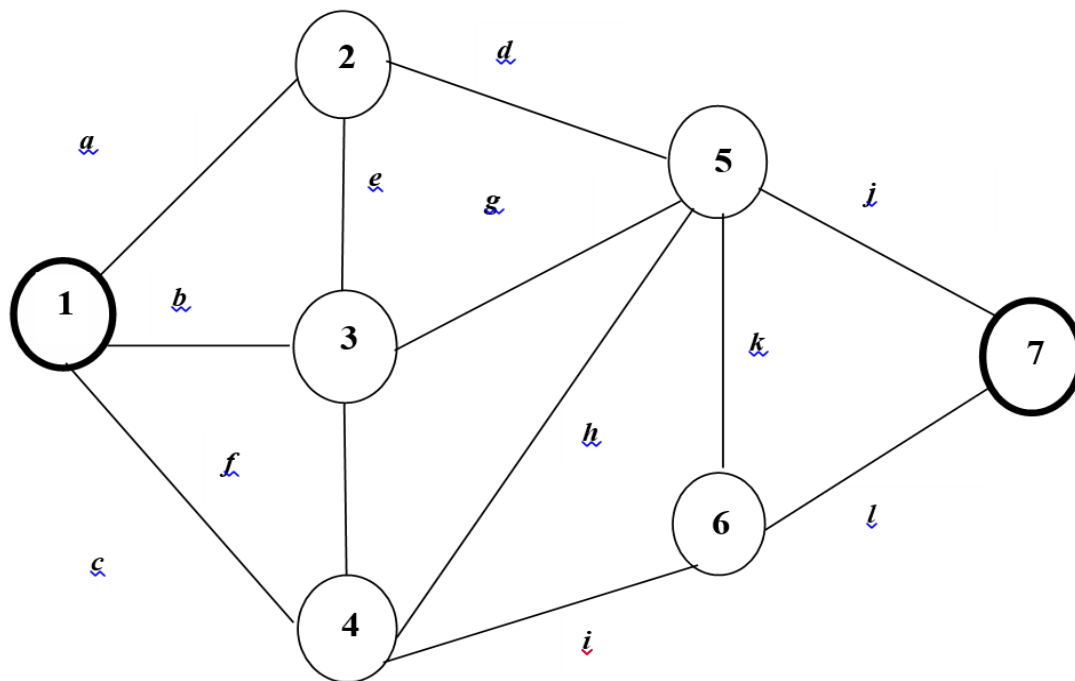


Figure 1.2: Air route network model showing connected nodes and distances

Every node has a range value tagged with a predecessor and a position. This clearly identifies the node as an airport in the flight pilot's graph. The range of a node is the shorter route from the origin node; and the node predecessor is the node preceding the given node in the fastest route from the source node. The node position may be permanent 'P' or temporary 'T'.

Keynote for the permanent labels of a node and for the provisional label of a node are used in this analysis. Attempting to make a permanent node implies it is included in the fastest route. When needed, the temporary node may be rewritten, but once a node becomes permanent it cannot be rewritten, except in second iterative traversal to determine the round trip.

### Network Model Routing and Implementation

Modified Dijkstra's algorithm whose strength depends on Comparison-Addition Model (CAM) and Bellman- Ford algorithm whose greatest advantage is the Relaxation Principle (RP) was used in this research for the determination of the shortest path in air route network model to develop an Intelligent Hybrid Model (IHM). A topological survey was made within the network system to determine the following parameters:

- i. Number of nodes in the network

- ii. Physical topology of the network
- iii. The link (edge) distance between a node in the network and its predecessor
- iv. The source node
- v. The destination node
- vi. The status of a node (either temporary or permanent)
  - i. Vertices configuration and classification

The above named parameters from the air route network system were used to develop the IHM using the following keynotes:

- (i) TL = temporary label of a node
- (ii) PL = permanent label of a node
- (iii) □ = permanent label of a node
- (iv) O = temporary label of a node
- (v) \* = permanent label of a node
- (vi) n = a node in the network *i.e* n<sub>1</sub> = node1, n<sub>2</sub> = node2, n<sub>3</sub> = node3, n<sub>4</sub> = node4, n<sub>5</sub> = node5 and n<sub>6</sub> = node6, n<sub>7</sub> = node7.
- (vii) d<sub>ij</sub> = distance cost between node *i* and *j* in the network
- (viii) a-l = edge weight (integer varies)

**CA-RP Optimization Analysis:**

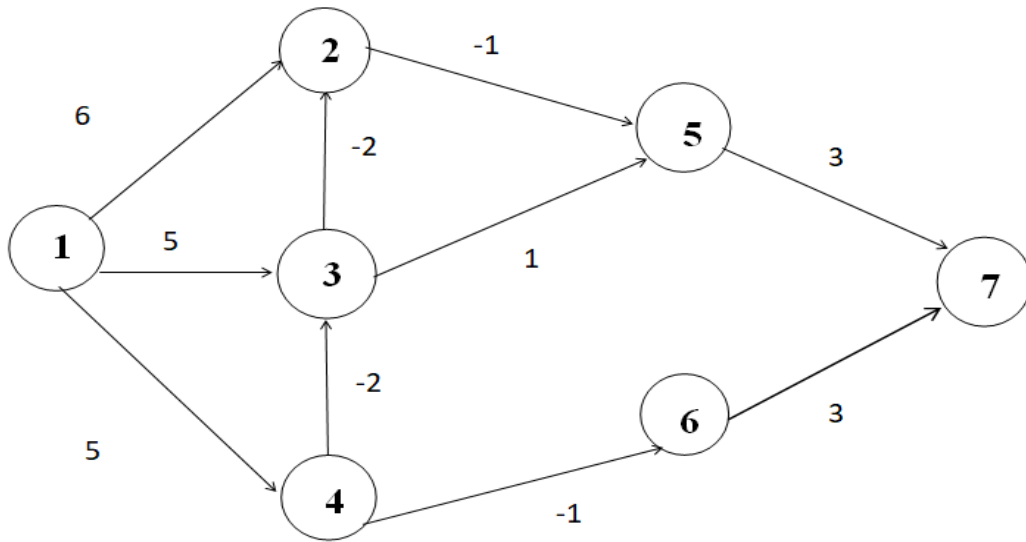


Figure 1.3: IHM using Relaxation Principle (RP)

**2. RP Analysis and Formula Derivation**

For dynamic programming strategy using Bellman-Ford, relaxation must be completed for all possible edges  $n-1$  times adhering to the following steps.

1. *Formulation Derivation*

Given:

Relax edges  $n-1$  of  $|V|$

If  $(u, v)$  is the edge size and distance between two vertices respectively; then the formula for relaxation is:

$(d[u] + c(u,v) < d[v])$  - Equation 1.1

$d[v] = d[u] + c(u,v)$  - Equation 1.2

2. *Select the edges*

List of edges is given thus given that the network is a digraph:

- $(1, 2) = |1, 2| = 6$
- $(1, 3) = |1, 3| = 5$
- $(1, 4) = |1, 4| = 5$
- $(2, 5) = |2, 5| = -1$
- $(3, 2) = |3, 2| = -2$
- $(3, 5) = |3, 5| = 1$
- $(4, 3) = |4, 3| = -2$

$$(4, 6) = |4, 6| = -1$$

$$(5, 7) = |5, 7| = 3$$

$$(6, 7) = |6, 7| = 3$$

3. Relaxation Iteration (n-1 times)

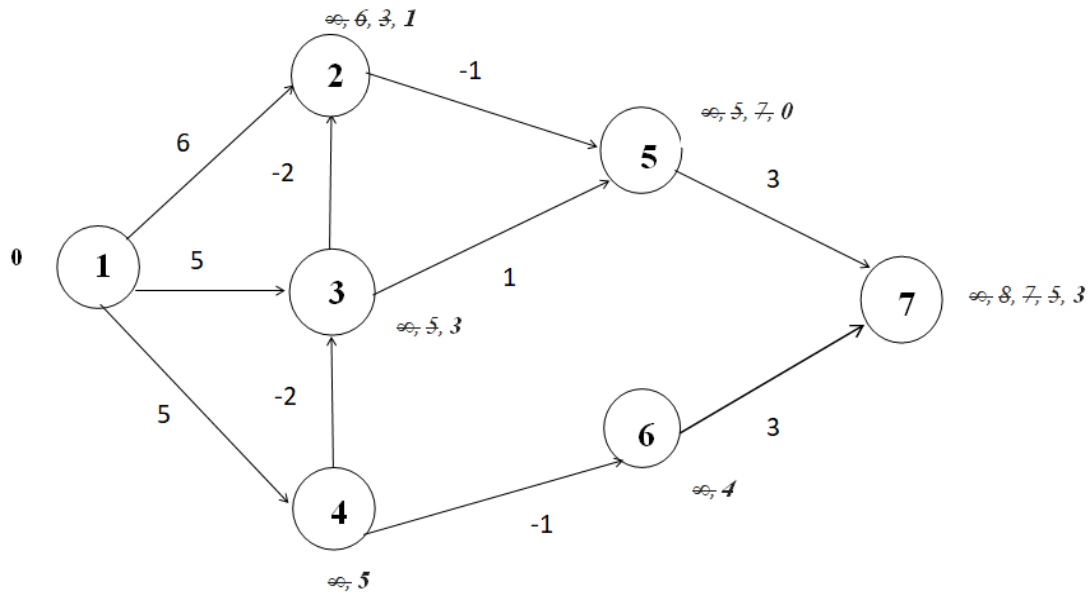


Figure 1.4: IHM Relaxed Edges and Vertices

4. Compute new edges and vertices weight

- $|V_1| = 0$
- $|V_2| = 1$
- $|V_3| = 3$
- $|V_4| = 5$
- $|V_5| = 3$
- $|V_6| = 4$
- $|V_7| = 3$

Time complexity:

$$\text{Number of possible relaxation} = O(|E| |V| - 1)$$

$$O(|V_n| |E_n|)$$

$$O(n^2)$$

Therefore;

$$\text{Number of edges} = \frac{n(n-2)}{2} = E \quad \text{Equation 4.3}$$

$$\text{Complete graph} = O(n^3), n^3 \text{ times} \quad \text{Equation 4.4}$$

### Updating Distance Values

Taking into account the combination of the algorithms and the distance price, the  $i^{th}$  will be the index of the current node. The algorithm defined the array  $j$  of temporary-labeling nodes from the present node  $i$  through a reference  $(ij)$  modified the distance value of the nodes. The range function  $dj$  of the node  $j$  is modified as follows for each  $j \in t$ : New  $dj = \min \{dj, di + dij\}$

Where  $dij$  is the network issue price connection  $i j$ )

Identify a  $j$  node with the smallest separation price  $dj$  among the nodes  $j \in J$ , find  $j^*$  such that  $\min dj = dj^*$

Switch the node tag  $j^*$  to permanent and make this node the new node.

### Termination

The algorithm definition updated by CM-RP framework remains efficient if when all nodes reachable from node S (source node) are permanently marked then avoid signaling implementation. If the temporary labeled node cannot be reached from the current node, then all the temporary labels becomes permanent, which signify completion.

### Presentation of Results

The optimal flow path for the modeled graph G is given in the figure 4.3. The diagram G contains the two sets  $v$  and  $E$ , where  $V$  is the array of vertices  $v_0, v_1, \dots, v_{n-1}$ , which are sometimes considered nodes, and  $E$  is the edge selection,  $e_1, e_2, \dots, e_n$ , where two nodes are connected. Mathematically, this state may be;  $G = (V, E)$

$$V(G) = (v_0, v_1, \dots, v_{n-1}) \text{ or set } \epsilon \text{ of vertices}$$

$$E(G) = (e_1, e_2, \dots, e_n) \text{ of set } \epsilon \text{ of edges}$$

### Conclusion

The study was motivated by the importance and necessity of efficient integration of data routing,

coordination and easy dispatch within connecting terminals which is used in many real-world scenarios. The algorithm was evaluated and technically experimented, and it was noticed that implementing the shortest path with the CM-RP framework will efficiently address the issue exiting mechanism and also, the deployment of Comparison-Additional Model (CAM) and Relaxation Principle (RP) in cooperating the multiple parameters like: permutation, combination and evaluation matrix will enhance its functionality across board.

One of the main aims or goals of any airline, transport and data delivery company is to maximize profit, while saving travelers time. An efficient and scheduled air routing coupled with delivery system is the key to meeting the ever-increasing air transportation user's demand especially in this rapid evolving globe. Bearing in mind the vast technological advancement in the aviation sector and the tremendous demands and increment of air users, it is super important to automate air routing, hence choosing shortest path to destination where applicable. This will in turn reduce consumption of fuel and carbon emission thereby ensuring great improvement in travel time while leave traveler with the concert time summation for a round trip.

### REFERENCE

- Aakanksha Gaur (2025). The Evolution of Transportation Networks: The Future is Sustainable - Means and Models. T. Editors of Encyclopaedia, March 22, 2025. Encyclopaedia Britannica. <https://www.britannica.com/science/geography/transportation>. Retrieved on Tuesday, April 29, 2025. 16:04:36.
- Arif, S., Atkin, J., & De Maere, G. (2003). A brief overview on an air transportation system (ATS). <https://www.sciencedirect.com/topics/engineering/air-transport>
- Borgwardt, K. M., & Kriegel, H. P. (2005). Shortest-path kernels on graphs. In Fifth IEEE

international conference on data mining (ICDM'05) (pp. 8-pp). IEEE.

Britannica, T. Editors of Encyclopaedia (2021). Transportation and their means.

Encyclopedia Britannica.  
<https://www.britannica.com/science/geography/transportation>

Budd, L., & Ison, S. (2020). An introduction to air transport management. In Air Transport

Management (pp. 29-34). Routledge.

Carlos Lopez (2019). Air Transport, A vital Challenge for Africa (Complex Journeys,

Limited Connections and Coverage) – African Air Transport Faces Many Challenges. How Can Its Development Be Nurtured?

Chedjou, J. C., & Kyamakya, K. (2015). A universal concept for robust solving of shortest

Curtin, K. M. (2007). Network analysis in geographic information science: Review,

assessment, and projections. Cartography and geographic information science, 34(2), 103-111.

Dreamstime (2025). Air Flight Route Illustration and Vectors Map. Flight Route Illustrations

& Vectors.  
<https://www.dreamstime.com/illustration/flight-route.html>. Retrieved on Friday, May 02, 2025. 15:02:45.

Duval, D. T. (2008). Aeropolitics, global aviation networks and the regulation of

international visitor flows. International business and tourism. Global issues, contemporary interactions, 91-105.

Eneh, A. H., Arinze, U. C (2017). Comparative Analysis and Implementation of Dijkstra's

Shortest Path Algorithm for Emergency Response and Logistic Planning. Nigerian Journal of Technology (NIJOTECH) Vol. 36, No. 3, July 2017, pp. 876 – 888.  
<http://dx.doi.org/10.4314/njt.v36i3.30>

Esuabana Ita Micah, Ikpang Ikpang Nkereuwem, & Okon Ekom-obong Jackson. (2016).

Shortest Transportation Route Network in Nigeria Using Floyd- Warshall's Algorithm.

Federal Airports Authority of Nigeria, FAAN (n.d). Aviation Sector Hand book and Reports

(2010–2018, 2019, Q3-Q4 of 2020, Q1 of 2021. Retrieved on Wednesday, October 13, 2021.

Floyd, R., (1962). Dynamic Programming Methods for Shortest Paths, Operations Research

Journal, vol. 2, no. 4, pp. 155–161, Oct. 1962.

Follesdal, Andreas; Wessel, Ramses; Wouters, Jan (2008). Multilevel Regulation and the EU:

The Interplay Between Global, European and National Normative Processes. The Netherlands: Brill. p. 187. ISBN 978-90-04-16438-3.

Hadi, H. M., & Ibrahim, I. M. (2025). A Comprehensive Review of Shortest Path Algorithms

for Network Routing. Asian Journal of Research in Computer Science, 18(3), 152–175.

<https://doi.org/10.9734/ajrcos/2025/v18i3584>

Ileoje, N. P. (2003). A New Geography of Nigeria Development. Fifth edition, Ikeja, Lagos:

Longman Nigeria Plc. Pp 1-257.

Javaid, M. A., (2013). Understanding Dijkstra's Algorithm. SSRN Electronic Journal.

10.2139/ssrn.2340905.2013. Member Vendor Advisory Council, CompTIA

Kairanbay Magzhan and Hajar Mat Jani (2013). A Review and Evaluations of Shortest Path

Algorithms. International Journal of Scientific & Technology Research • January 2013 - at:

<https://www.researchgate.net/publication/310594546>. Retrieved on Wednesday, October 13, 2021

Kerur, P., & Chakrasali, R. L. (2017). Optimal path for power flow in future distribution

system planning with uninterruptable power supply using graph theory. *Int. J. Electron. Elect. Comput. Syst.*, 6, 24-32.

Kinza Yasar, (n.d). Concepts of Routing Table in Air Route Network.

<https://www.techtarget.com/searchnetworking/definition/routing-table>. Retrieved on 28/02/2023.

Kleinberg, Jon and Tardos, Éva. (2006). *Algorithm Design*. Boston, MA: Pearson Publisher:

Addison-Wesley. Pearson Education, Inc.

Ladan, S. I. (2012). An analysis of air transportation in Nigeria. *Journal of research in*

national development, 10(2).

Lavinskaya, O. Y., Kurchenkova, T. V., & Kuripta, O. V. (2020). Shortest path algorithm for

graphs in instances of semantic optimization. In *Journal of Physics: Conference Series* (Vol. 1479, No. 1, p. 012036). IOP Publishing.

Lee, B., Kim, H., and Park, J., (2022). A Hybrid Shortest Path Algorithm Combining A\* with

Swarm Intelligence Heuristic for VANETs," in *Proc. Veh. Technol. Conf. (VTC), 2022*, pp. 456–462.

Lewis, W. M. (1936). The Significance of Transportation to Civilization. *The Annals of the*

*American Academy of Political and Social Science*, 187, 1–6. <http://www.jstor.org/stable/1019605>

Li, Q., Chen, B. Y., Wang, Y., & Lam, W. H. (2015). A hybrid link-node approach for

finding shortest paths in road networks with turn restrictions. *Transactions in GIS*, 19(6), 915-929.

Li, S., Zhang, H., Li, Z., & Liu, H. (2021). An Air Route Network Planning Model of

Logistics UAV Terminal Distribution in

Urban Low Altitude  
Airspace. *Sustainability*, 13(23), 13079.  
<https://doi.org/10.3390/su132313079>

Liu, F., Wang, M., and Xu, S., (2023). GPU-Accelerated Shortest Path Algorithm Designed

for real-time Applications in Smart Cities, *IEEE Access*, vol. 11, pp. 3324–3335, 2023.

Magzhan, K., & Jani, H. M. (2013). A review and evaluations of shortest path algorithms. *Int.*

*J. Sci. Technol. Res.*, 2(6), 99-104.

Mba, U. E.; Mbeledogu, N. N.; Okpara C. (2026). Optimal Path System for Negative Edges in Single Source

Multiple Destination Networks using Intelligent Hybrid Model. PhD Thesis, Department of Computer Science, Nnamdi Azikiwe Univeristy, Awka, Nigeria.

Mba, U. E., & Mbeledogu, N. N., (2024). Shortest Path System for Nigerian Air Dispatch

Network Using Modified Dijkstra's Algorithm, *International Journal of Advanced Research in Computer and Communication Engineering*, Vol.13, Issue 2, Pgs. 171-181

McMahan, H. B., & Gordon, G. J. (2005). Generalizing Dijkstra's algorithm and Gaussian

elimination for solving MDPs. School of Computer Science, Carnegie Mellon University.

Natasha Sharma (2019). Importance of Distance Metrics in Machine Learning Modelling.

TDS Archive Publication. Medium. <https://medium.com/data-science/importance-of-distance-metrics-in-machine-learning-modelling-e51395ffe60d>. Retrieved on Thursday, July 10, 2025.

Nissalike Vivian (n.d). Aviation Benefits: contributing to global economic prosperity. The

first of a two-part series sharing excerpts from the Industry High-Level Group (IHLG) report, Aviation Benefits.

<https://unitingaviation.com/news/economic-development/aviation-benefits-for-a-better-future>

Rodrigue, Jean-Paul (2024). Transportation Modes, Modal Competition and Modal Shift. The

Geography of Transport Systems. New York: Routledge, 402 pages. ISBN 9781032380407. DOI:10.4324/9781003343196. <https://transportgeography.org/contents/chapter5/transportation-modes-modal-competition-modal-shift/>. Retrieved on Monday, April 21, 2025, 17:45:10.

Rowell, David (2022). "Freedoms of the Air". The Travel Insider. Retrieved 25 November 2025.

Roy, G., Sinha, A., and Das, P., (2023). A Hybrid Algorithm for Routing in MANETs

Combining Dijkstra's and Bellman-Ford," in Proc. Int. Conf. Mobile Ad Hoc Netw., 2023, pp. 78–85.

Saheed B, Gratton G. B, (2015). An approach to evaluate lift generated by an annular-

Coanda-wing for vertical/short take-off and landing applications, Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering, Vol: 226, Pages: 1298-1314, ISSN: 0954-4100

Salman Arif; Jason Atkin, and Geert De Maere (2003). A Brief Overview on an Air

Transportation System (ATS) - Department of Computer Science, University of Nottingham. Technical report, February 19, 2003. DOI: 10.13140/RG.2.2.14090.80329...<https://www.researchgate.net/publication/351885390>. Retrieved on Wednesday, October 13, 2021.

Shu-Xi, W. (2012). The improved Dijkstra's shortest path algorithm and its application.

Procedia Engineering, 29, 1186-1190.

Singh Atul (2024). Secrets of the Bellman-Ford Algorithm and Its Importance. Meduim

Publishers. February 3, 2024. Retrieved from "https://medium.com/@singhatul1155/secrets-of-the-bellman-ford-algorithm-why-dijkstras-algorithm-get-more-priority-over-bellman-ford-d460db32abd0", Sunday, June 1, 2025; 18:46:43.

Singh, I., Nair, P., and Patel, K., (2024). Real-time Shortest Path Algorithm for Intelligent

Transportation Systems using Deep Reinforcement learning," IEEE Trans. Intell. Transp. Syst., vol. 26, no. 1, pp. 112-123, Jan. 2024.

Singhal, A. (2020). Shortest Path Problem/Shortest Path Algorithms | Examples. Retrieved

from <https://www.gatevidalay.com/shortest-path-algorithms-shortest-path-problems>.

Sivakummar, S and Chandraskra M. (2022). Analysis of Dijkstra's Shortest Path Algorithm

with appropriate parameters. Analysis of Shortest Path Algorithms. International Journal of Engineering Applied Sciences and Technology, 2022

Sommer, C. (2010). Approximate shortest path and distance queries in networks.

Unpublished Doctor of Philosophy Thesis, University of Tokyo, Tokyo Japan.

Wang, D., Liu, X., and Zhao, Y., (2023). A Deep Learning-Based Approach for K-Shortest

Paths in large scale Road Networks using graph attention Networks. IEEE Trans. Intell. Transp. Syst., vol. 25, no. 3, pp. 467–478, Mar. 2023.

Wang, T. and Zhang, F. (2022). Attacking Random Forest Classifiers based on Shortest Path

Algorithm Asia Conference on Algorithms, Computing and Machine Learning (CACML), Hangzhou, China, 2022, pp. 193-199, doi: 10.1109/CACML55074.2022.00039.

keywords: {Machine learning algorithms; Input variables; Decision making; Training data; Forestry; Robustness; Classification

algorithms; adversarial machine learning; random forest; evasion attacks },

Wang, X. Z., (2018). The Comparison of Three Algorithms in Shortest Path Issue. First

International Conference on Advanced Algorithms and Control Engineering, IOP Conf. Series: Journal of Physics: Conf. Series, vol. 1087, no. 2, pp. 022011, 2018. doi:10.1088/1719-6596/1087/2/022011.

Weisstein, E. N. (2009). Weighted Graph. Wolfram Web Resources. Retrieved from

<http://mathworld.wolfram.com/weightedGraph.html>.

Xu, R., Zhou, H., and Zhang, Y., (2021). Reinforcement Learning for Adaptive Shortest Path

Routing in Complex Networks," IEEE Access, vol. 9, pp. 120164–120175, 2021.

Yang, X., and Medhi, D., (2010). Routing in

Network Virtualization: Enhancements and

Challenges," IEEE Communication. Mag., vol. 48, no. 7, pp. 128–135, Jul. 2010.

Zhan, F. B. (1997). Three fastest shortest path algorithms on real road networks: Data

structures and procedures. Journal of geographic information and decision analysis, 1(1), 69-82.

Zhang, C., Li, F., and Wong, K., (2023). Multi-Objective Optimization Framework for

Shortest Path Routing in IoT Networks," IEEE Internet Things J., vol. 9, no. 2, pp. 78–89, Feb. 2023.

Zhu, A., & Pan, W. (2022). An innovative crane-lift path planning system for high-rise

modular integrated construction. Construction Robotics, 6(2), 133-150.