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Further Contribution to the Square Transformation of the Error Component of the Multiplicative Time Series Model

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Abstract | **Original Research Article**

Conditions initially designed to yield successful transformation in time series modeling are considered. For these cases, this article proposes an approach aimed at exploring the range at which the expectation of the untransformed attains equilibrium with the transformed and maintains variance as a proportional product of the untransformed. Methodologically, new results are obtained, and relations in comparison of normality are compared. The square transformation has a moderate effect on distribution shape. The square transformed could be used to reduce left skewness as squaring makes sense even if the variable concerned is zero or positive, given that and are identical. It is the motive of this paper to primarily investigate the limits of the square transformation with a view to eliminating distributional assumptions that may impede expectations for the confidence intervals. We compared means and variances of both the transformed and untransformed to establish the bounding curves.

Keywords: Transformation, Statistical properties, distributional assumptions, confidence intervals.

1. INTRODUCTION

The continuous random variable x takes on all real values and has mean μ and variance $\sigma^2 > 0$. Many statistical techniques in both the applied and theoretical settings are based on the classical normal distribution. The plot of the normal density function for several values of σ^2 holding μ constant, reveals the shapes of the curves will vary the degrees of its flatness increasing with increased values of σ^2. If on the other hand, the value of μ is varied with σ^2 being fixed, the shapes of the curves will remain the same with only shifting of the curves to either sides, left or right. Thus the value of μ determines the centre of the probability density function while σ^2 determines the dispersion or spread.

If X has the normal distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad ; \qquad -\infty < x < \infty, \quad \sigma > 0 \tag{1}$$

In some situation, we may wish to partition the domain for one reason or the other so we do not want to admit of certain values of X, [13] shows we cut such values off and amend or normalize the distribution to accommodate what has been done. The transformed distribution in probability and statistics is known as the truncated normal distribution. The properties of the truncated distributions for various families of probability densities have been discussed in the literature. In this study, we first derived a transformation of the normal population which accounts for a probability inequality, and then using the transformation, obtain a property of the mean and variance of the subpopulation.

Truncation can be carried out on either side of $x \in \Re$ [9] and has wide applications in statistics and econometrics, for example, it is used to model the probabilities of the binary outcomes in the Probit model and to model censored data in the Tobit model.

A variety of transformation functions on the error component of the multiplicative times model have been explored. Some of the transformations include

2. Preliminaries

2.1. Definition

Suppose $X \sim N(\mu, \sigma^2)$ has a normal distribution and lies within the interval $X \in (a,b)$, $-\infty \le a < b \le \infty$. Then X conditional on $a < X < b$ has a truncated normal distribution. Its probability density function, f, for $a \le x \le b$, is given by

$$f(x;\mu,\sigma,a,b) = \frac{\frac{1}{\sigma}\varphi\left(\frac{x-\mu}{\sigma}\right)}{\Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)} \qquad 2.$$

Where

$\varphi(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z^2\right)$ is the probability density function of the standard normal distribution and $\Phi(.)$ is the cumulative distribution function. There is an understanding that if $b = \infty$, then $\Phi\left(\frac{b-\mu}{\sigma}\right) = 1$, and similarly, if $a = -\infty$, then $\Phi\left(\frac{a-\mu}{\sigma}\right) = 0$

A truncated distribution is ultimately the part of an untruncated distribution that is above or below some specified value therefore. The mean of a truncated distribution is larger than that of the original distribution. However, the variance is smaller [6].

A time series is a set of values observed sequentially through time. The series may be denoted by $X_1, X_2, ..., X_t$, where t refers to the time period and X refers to the value. If the $X's$ are exactly determined by a mathematical formula, the series is said to be *deterministic*. If future values can be described only by their probability distribution, the series is said to be a *statistical* or *stochastic* process. Long-term time series, tend to include trend, seasonal and cyclical patterns; therefore, requires more sophisticated forecasting methods. These methods can be classified broadly as trend - seasonal and Box-Jenkins time series analysis. Trend – seasonal analysis uses decomposition approach. Here, time series components are isolated in terms of

(i) **Trend** (T_t) - A long-term movements in the mean;
(ii) **Seasonal effects** (I_t) - A cyclical fluctuations related to the calendar;
(iii) **Cycles** (C_t) - Other cyclical fluctuations (such as business cycles);
(iv) **Residuals** (E_t) - other random component error term).

[4] time series analysis is akin to multiple regression analysis in which some of the predictor variables are lagged response variables, lagged residuals from a fit of lagged response variables or both lagged response variables and lagged residuals. Thus, [2] application to time series analysis captures time series in its own special way.

Time Series Models

A time series model for the observed data (X_t) is a specification of the joint distributions (or possibly only the means and covariances) of a sequence of random variables $\{X_t\}$ of which $\{X_t\}$ is postulated to be a realization.

The three major models used in descriptive time series are fashioned in that the idea is to create separate models for the four variable- components (T_t, I_t, C_t, E_t) and then combine them to form the following models:

(i) Multiplicative model, where
$$X_t = T_t..I_t.C_t.E_t$$

(ii) Additive model
$$X_t = T_t + I_t + C_t + E_t$$

(iii) Mixed model
$$X_t = T_t.I_t.C_t + E_t$$

The choice of any model is ascertained by plotting the graph of periodic mean and standard deviation of the data from the Buys Ballot table. Thereafter, the direction of the plots is examined to determine which of the model is most appropriate for the time series decomposition.

The Use of time Plot

Time plot enhances visualization of important features inherent in a series such as Trend, Seasonality, Discontinuities and Outliers. Plotting the data may indicate if it is desirable to transform the values of the observed variable. Such result would to choosing an appropriate model that best represents those factors that were discovered during the plot. Moreso, [1], [2], and [11] made specific conditions and recommendations on choice and caution in data transformations, however, these works did not consider the implications on transformations with respect to the multiplicative time series model until [5] pioneered a work on the implications of logarithmic transformations on the error component of the multiplicative model. His work spurred scholars to explore this area of time series analysis, eliciting contributions.

Need for Data Transformation

Transformation involves applying a mathematical function (e.g. squaring the data) to each data point. A transformation when the data is excessively skewed positively or negatively. The most frequent reason why researchers transform their data is to make the data “normal”, and thus fulfill one of the assumptions of conducting a parametric means comparison. Other reasons include informative graphs of the data, better outlier identification (or getting outliers in line), and increasing the sensitivity of statistical tests. [11] probably had the right idea when he called data transformation calculations “re-expressions” rather than “transformations”. A researcher is basically re-expressing what the data have to say in other terms.

A review of the literature shows that a considerable number of transformations of binomial, negative binomial, Poisson and χ^2 variables have been proposed. For more references see [1], [2]. Further, examples of stochastic modeling using the truncated normal random variable include the modeling of demand functions in the determination of safety stocks by [7] and [8].

Z Transformation

One of the major transformations in standard form is the z transformation. While the normal density has many desirable properties, it cannot be integrated in familiar terms [11] . If we wish to compute say,

$$P(a < X \leq b) = \int_a^b f(x)dx = \int_a^b \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \tag{6}$$

A first step in dealing with such an integral is to bring the integrand to a standard form through the simple substitution

$$z = \frac{x-\mu}{\sigma} \tag{7}$$

Which is the z-transformation for nonnegative random variables, as the substitution reduces

$X \sim N(\mu, \sigma^2)$ to $Z = \frac{x-\mu}{\sigma} \sim N(0,1)$

The standard normal has the cdf

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \tag{8}$$

Takes values in [0,1] with typical values as
$\Phi(-\infty) = 0,\ \Phi(0) = .5,\ \Phi(1) = .841,\ \Phi(2) = .977,\ \Phi(3) = .9987,\ \Phi(\infty) = 1.$

Theorem: If $X \sim N(\mu, \sigma^2)$, then the random variable $Z = \frac{x-\mu}{\sigma} \sim N(0,1)$

Proof: We show that Z is standard normal by finding the probability density function of Z, using the cumulative distribution function approach.

$$F(z) = P(Z \leq z)$$

$$= P\left(\frac{X-\mu}{\sigma} \leq z\right)$$

$$= P(X \leq \sigma z + \mu)$$

$$= \int_{-\infty}^{\sigma z+\mu} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \int_{-\infty}^{z} \frac{1}{\sigma\sqrt{2\pi}} \sigma e^{-\frac{1}{2}w^2} dw, \qquad \text{where } w = \frac{x-\mu}{\sigma},$$

Hence $$f(z) = F\ (z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \quad (9)$$

3. Methodology

The probability density function of the n^th^ power transformation according to [3] is given by

$$f(y) = \frac{|\frac{1}{n}|\, y^{\frac{1}{n}-1} e^{-\frac{1}{2}\left(\frac{y^{\frac{1}{n}}-1}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)}$$

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The mean and variance for $n = 2$ which yields the square transformation:

$$E(y) = 1 + \frac{n\sigma}{\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)} + \frac{n(n-1)\sigma^2}{2!\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)}\left(\frac{-e^{-\frac{1}{2}\sigma^2}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1+\Pr(\chi^2_{(1)} \le \frac{1}{\sigma^2})\right]\right) +$$
$$\frac{2n(n-1)(n-2)\sigma^3}{3!\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)}\left[1+\frac{1}{2\sigma^2}\right]e^{-\frac{1}{2}\sigma^2}$$

11

$n = 2,$ we have

$$E(y) = 1 + \frac{2\sigma e^{-\frac{1}{2}\sigma^2}}{\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)} + \frac{2(2-1)\sigma^2}{2!\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)} + \left(\frac{-e^{-\frac{1}{2}\sigma^2}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1+\Pr(\chi^2_{(1)} \le \frac{1}{\sigma^2})\right]\right) +$$
$$\frac{2(2)(2-1)(2-2)\sigma^3}{3!\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)}\left[1+\frac{1}{2\sigma^2}\right]e^{-\frac{1}{2}\sigma^2}$$

$$= 1 + \frac{2\sigma e^{-\frac{1}{2}\sigma^2}}{\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)} - \frac{\sigma e^{-\frac{1}{2}\sigma^2}}{\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)} + \frac{\sigma^2}{2\left(1-\phi(-\frac{1}{\sigma})\right)}\left[1+\Pr(\chi^2_{(1)} \le \frac{1}{\sigma^2})\right]$$

$$= 1 + \frac{\sigma e^{-\frac{1}{2}\sigma^2}}{\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)} + \frac{\sigma^2}{2\left(1-\phi(-\frac{1}{\sigma})\right)}\left[1+\Pr(\chi^2_{(1)} \le \frac{1\sigma^2}{})\right]$$

12

$$E(y^2)=1+\frac{2n\sigma e^{-\frac{1}{2}\sigma^2}}{\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)}+\frac{2n(2n-1)\sigma^2}{2!\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)}\left(\frac{-e^{-\frac{1}{2}\sigma^2}}{\sigma}+\frac{\sqrt{2\pi}}{2}\left[1+\Pr(\chi^2_{(1)}\le\frac{1}{\sigma^2})\right]\right)+$$
$$\frac{2(2n)(2n-1)(2n-2)\sigma^3}{3!\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)}\left[1+\frac{1}{2\sigma^2}\right]e^{-\frac{1}{2}\sigma^2} \quad 13$$

$n=2,$

$$E(y^2)=1+\frac{2(2)\sigma e^{-\frac{1}{2}\sigma^2}}{\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)}+\frac{2(2)\left(2(2)-1\right)\sigma^2}{2!\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)}\left(\frac{-e^{-\frac{1}{2}\sigma^2}}{\sigma}+\frac{\sqrt{2\pi}}{2}\left[1+\Pr(\chi^2_{(1)}\le\frac{1}{\sigma^2})\right]\right)+$$
$$\frac{2\left(2(2)(2(2)-1\right)(2(2)-2)\sigma^3}{3!\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)}\left[1+\frac{1}{2\sigma^2}\right]e^{-\frac{1}{2}\sigma^2}$$

$$=1+\frac{4\sigma e^{-\frac{1}{2}\sigma^2}}{\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)}+\frac{4(3)\sigma^2}{2!\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)}\left(\frac{-e^{-\frac{1}{2}\sigma^2}}{\sigma}+\frac{\sqrt{2\pi}}{2}\left[1+\Pr(\chi^2_{(1)}\le\frac{1}{\sigma^2})\right]\right)+$$
$$\frac{2(4)(3)(2)\sigma^3}{3!\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)}\left[1+\frac{1}{2\sigma^2}\right]e^{-\frac{1}{2}\sigma^2}$$

$$=1+\frac{4\sigma e^{-\frac{1}{2}\sigma^2}}{\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)}+\frac{6\sigma^2}{\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)}\left(\frac{-e^{-\frac{1}{2}\sigma^2}}{\sigma}+\frac{\sqrt{2\pi}}{2}\left[1+\Pr(\chi^2_{(1)}\le\frac{1}{\sigma^2})\right]\right)+$$
$$\frac{8\sigma^3}{\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)}\left[1+\frac{1}{2\sigma^2}\right]e^{-\frac{1}{2}\sigma^2}$$

$$=1+\frac{4\sigma e^{-\frac{1}{2}\sigma^2}}{\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)}-\frac{6\sigma^2}{\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)}+\frac{3\sigma^2}{\left(1-\phi(-\frac{1}{\sigma})\right)}\left(\left[1+\Pr(\chi^2_{(1)}\le\frac{1}{\sigma^2})\right]\right)+\frac{8\sigma^3 e^{-\frac{1}{2}\sigma^2}}{\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)}+$$
$$\frac{4\sigma e^{-\frac{1}{2}\sigma^2}}{\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)}$$

$$E(y^2) = 1 + \frac{2\sigma e^{-\frac{1}{2}\sigma^2}}{\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)} + \frac{3\sigma^2}{\left(1-\phi(-\frac{1}{\sigma})\right)}\left[1+\Pr(\chi^2_{(1)} \le \frac{1}{\sigma^2})\right] + \frac{8\sigma^3 e^{-\frac{1}{2}\sigma^2}}{\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)} \qquad 14$$

$$Var(y) = E(y^2) - (E(y))^2$$

$$= 1 + \frac{2\sigma e^{-\frac{1}{2}\sigma^2}}{\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)} + \frac{3\sigma^2}{\left(1-\phi(-\frac{1}{\sigma})\right)}\left[1+\Pr(\chi^2_{(1)} \le \frac{1}{\sigma^2})\right] + \frac{8\sigma^3 e^{-\frac{1}{2}\sigma^2}}{\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)} -$$

$$\left[1 + \frac{\sigma e^{-\frac{1}{2}\sigma^2}}{\sqrt{2\pi}\left(1-\phi(-\frac{1}{\sigma})\right)} + \frac{\sigma^2}{2\left(1-\phi(-\frac{1}{\sigma})\right)}\left[1+\Pr(\chi^2_{(1)} \le \frac{1}{\sigma^2})\right]\right]^2$$

[10] show for values of $\sigma = 0.01, 0.011, 0.012, \ldots, 0.298. 0.299, 0.300$ that $E(X) = E(Y) \cong 1.0$ to one decimal place for the interval $\{0 < \sigma \le 0.027 : b = 0.027\}$

Results

The table 1 below shows the transformed y and untransformed x variaates with the compared effectiveness and their bounds.

Table 1

Decimal Places	$E(X) = E(Y)$	$Vay(Y) = 4 * Var(X)$
4	$0 < \sigma < 0.003$	$0 < \sigma \le 0.002$
3	$0 < \sigma < 0.012$	$0 < \sigma \le 0.007$
2	$0 < \sigma < 0.040$	$0 < \sigma \le 0.023$
1	$0 < \sigma < 0.129$	$0 < \sigma \le 0.074$

Conclusion

The bounds where they hold are just within the normality regions, which explains that the transformation comes from $X \sim N(0,1)$. Error variance stabilization is used to treat data that are heteroscacedastic, having magnitude of variations (measurement errors) that change across the range of the data Transformation.

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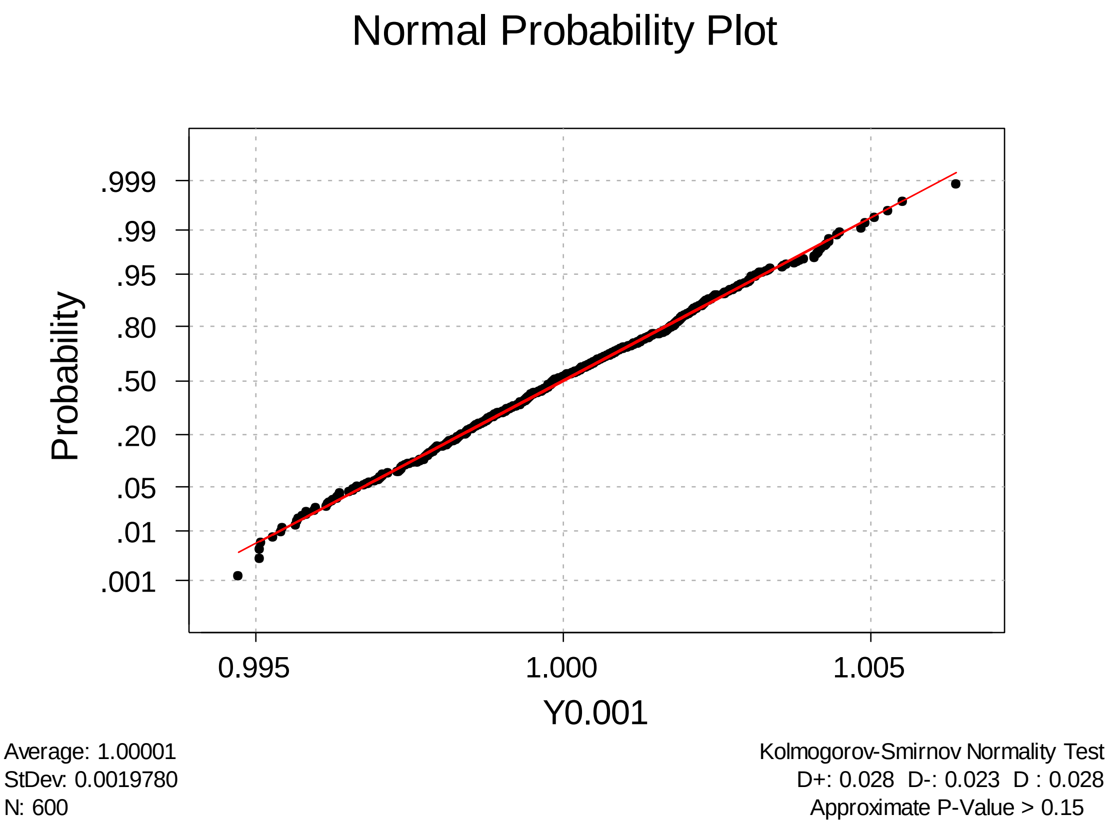
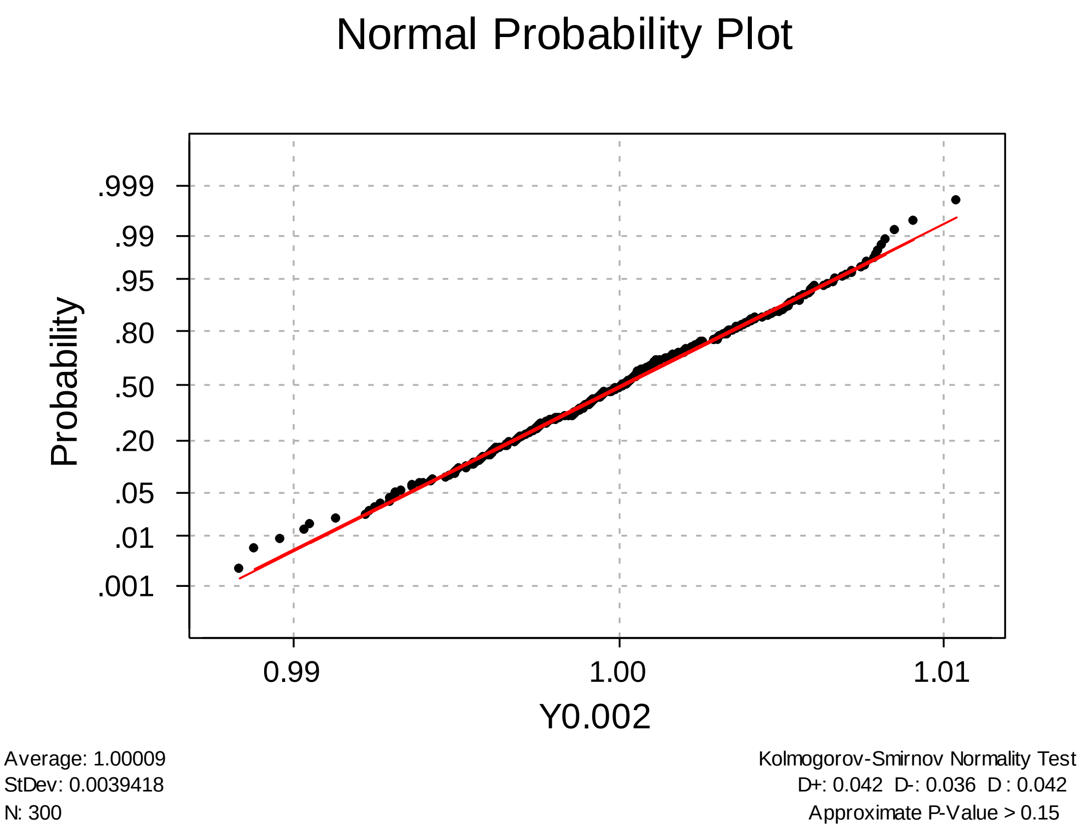

Normal Probability Plot
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
0.995
1.000
1.005
Y0.001
Average: 1.00001
StDev: 0.0019780
N: 600
Kolmogorov-Smirnov Normality Test
D+: 0.028 D-: 0.023 D : 0.028
Approximate P-Value > 0.15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Probability
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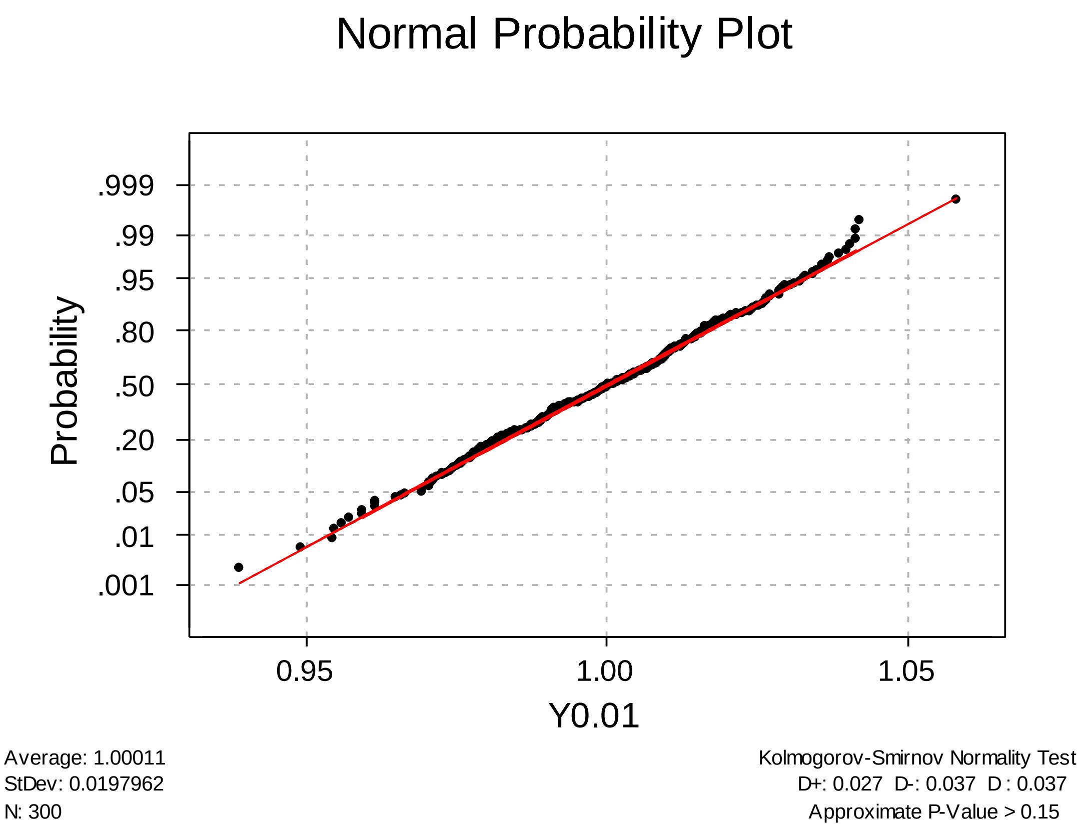
.99
.95
.80
.50
.20
.05
.01
.001
0.99
1.00
1.01
Y0.002
Average: 1.00009
StDev: 0.0039418
N: 300
Kolmogorov-Smirnov Normality Test
D+: 0.042 D-: 0.036 D : 0.042
Approximate P-Value > 0.15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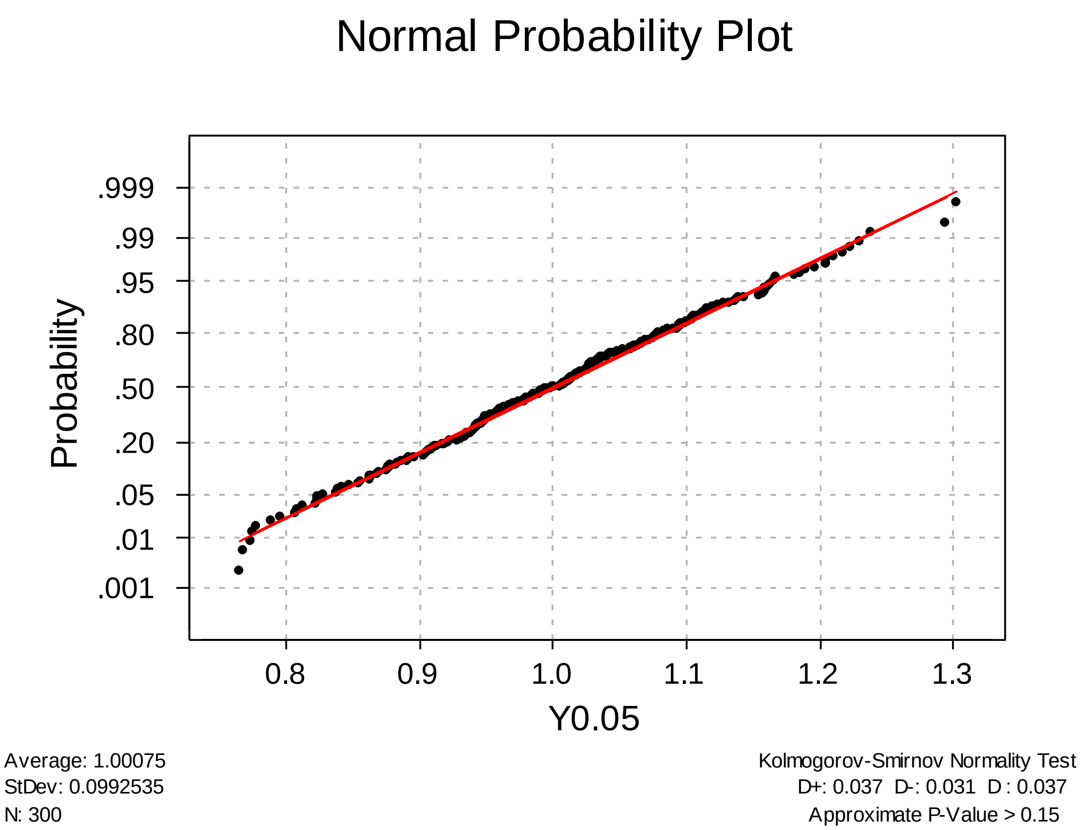
.999
.99
.95
.80
.50
.20
.05
.01
.001
Probability
0.95
1.00
1.05
Y0.01
Average: 1.00011
StDev: 0.0197962
N: 300
Kolmogorov-Smirnov Normality Test
D+: 0.027 D-: 0.037 D : 0.037
Approximate P-Value > 0.15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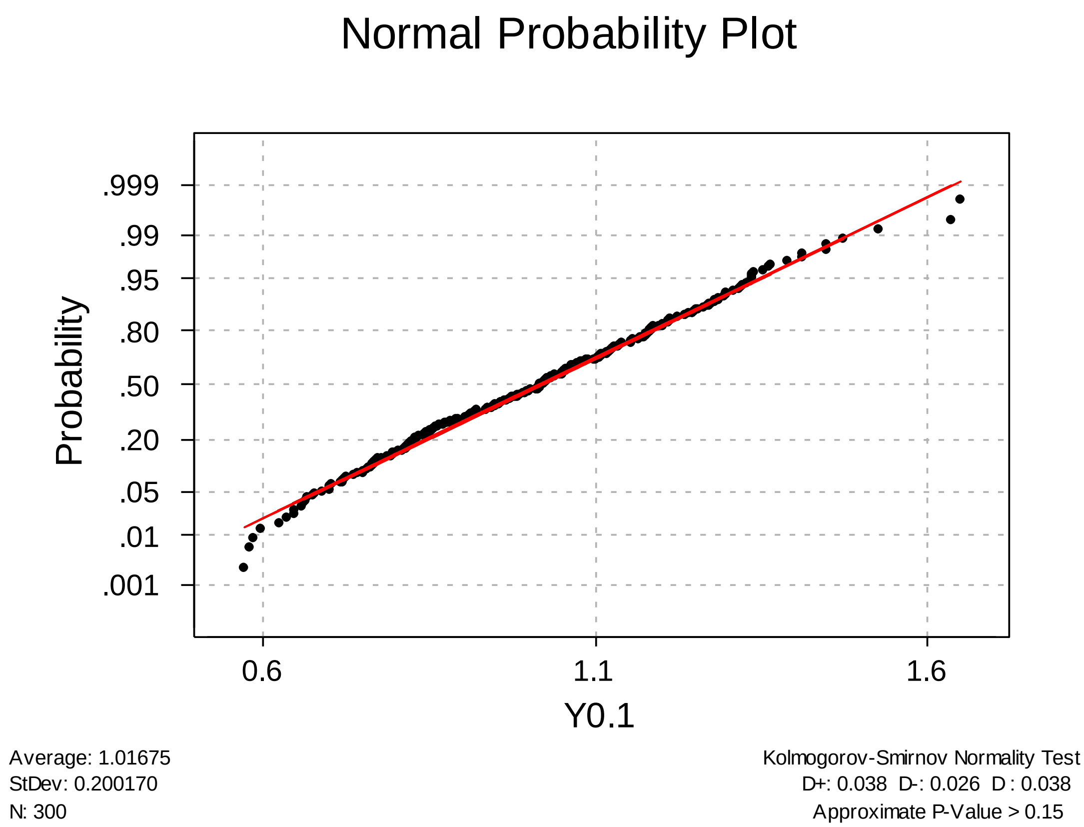
.99
.95
.80
.50
.20
.05
.01
.001
Probability
0.8
0.9
1.0
1.1
1.2
1.3
Y0.05
Average: 1.00075
StDev: 0.0992535
N: 300
Kolmogorov-Smirnov Normality Test
D+: 0.037 D-: 0.031 D : 0.037
Approximate P-Value > 0.15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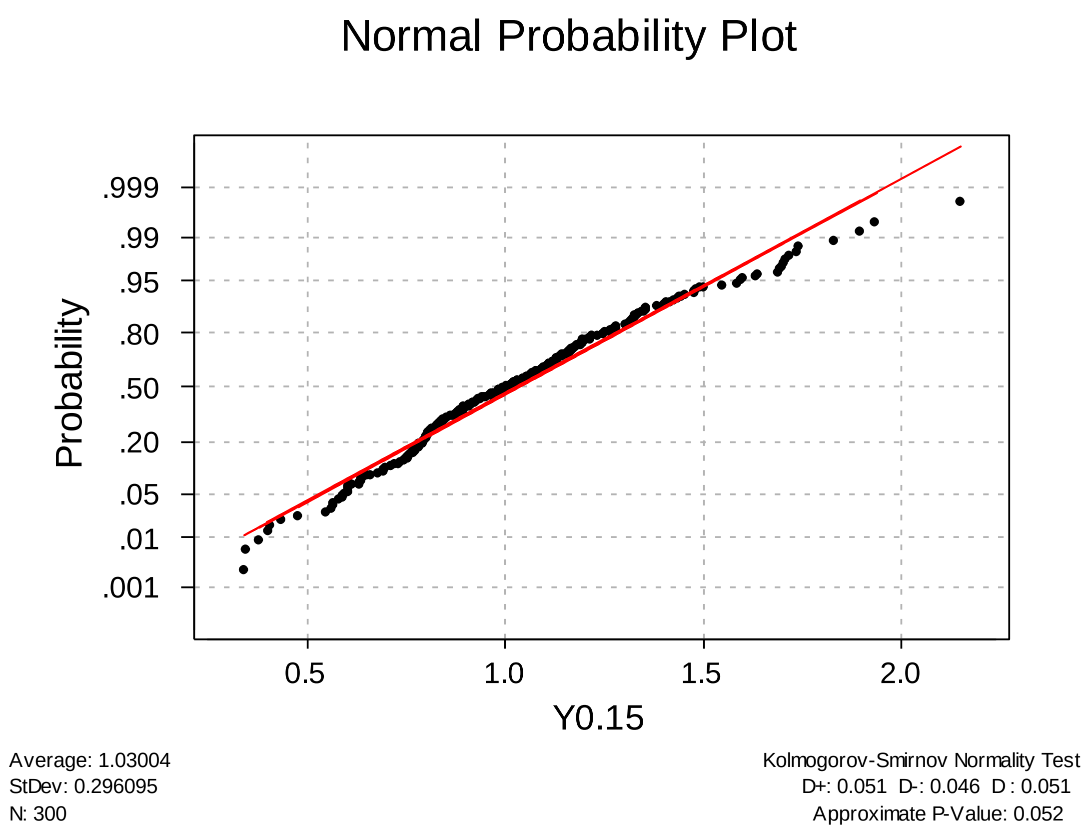
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
0.6
1.1
1.6
Y0.1
Average: 1.01675
StDev: 0.200170
N: 300
Kolmogorov-Smirnov Normality Test
D+: 0.038 D-: 0.026 D : 0.038
Approximate P-Value > 0.15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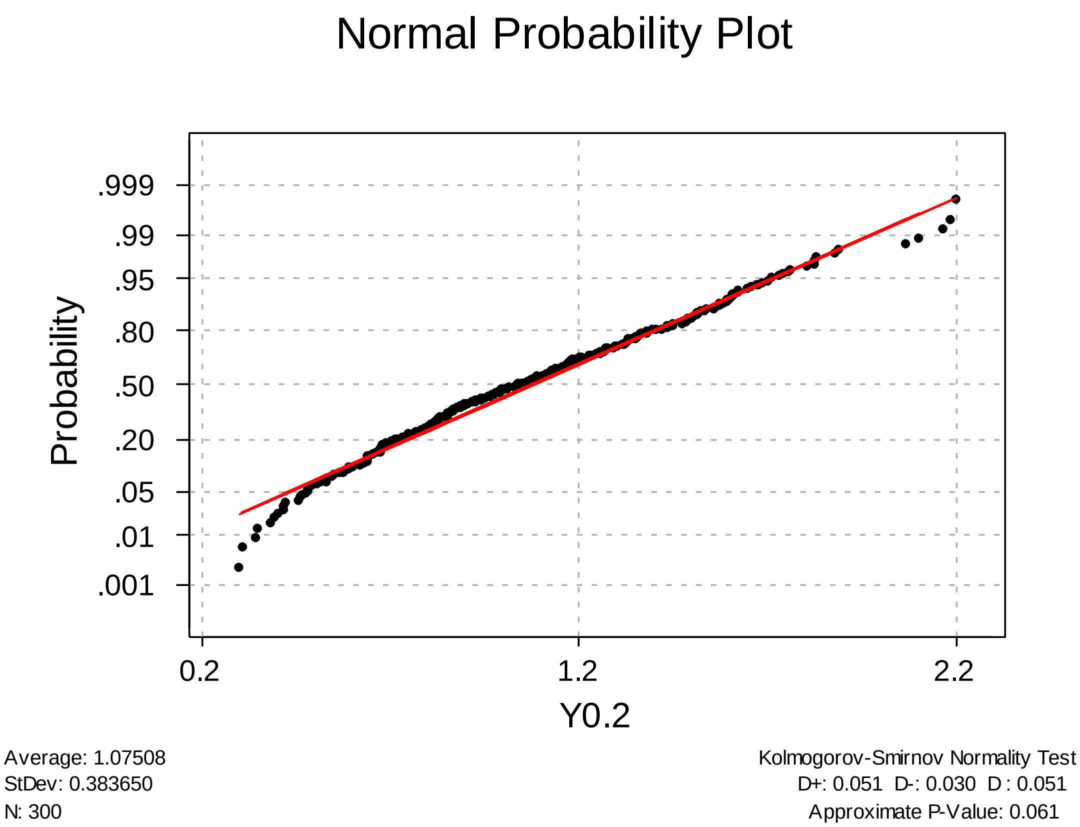
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
0.5
1.0
1.5
2.0
Y0.15
Average: 1.03004
StDev: 0.296095
N: 300
Kolmogorov-Smirnov Normality Test
D+: 0.051 D-: 0.046 D : 0.051
Approximate P-Value: 0.052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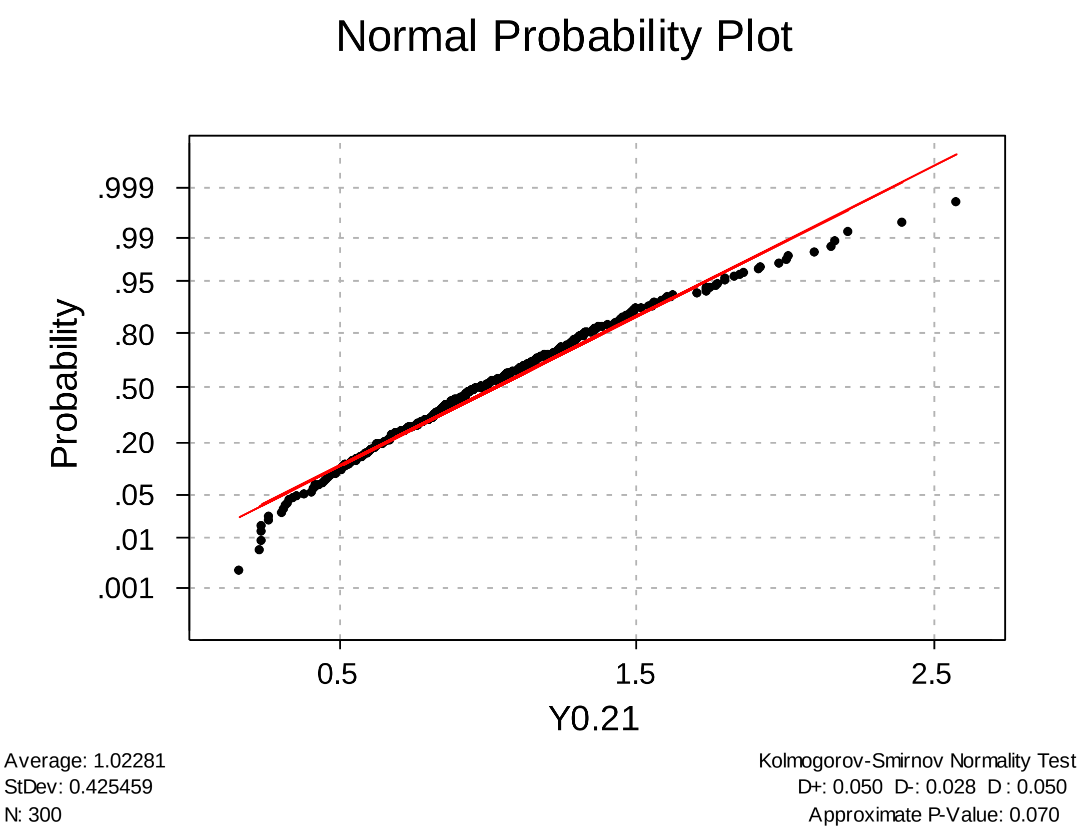
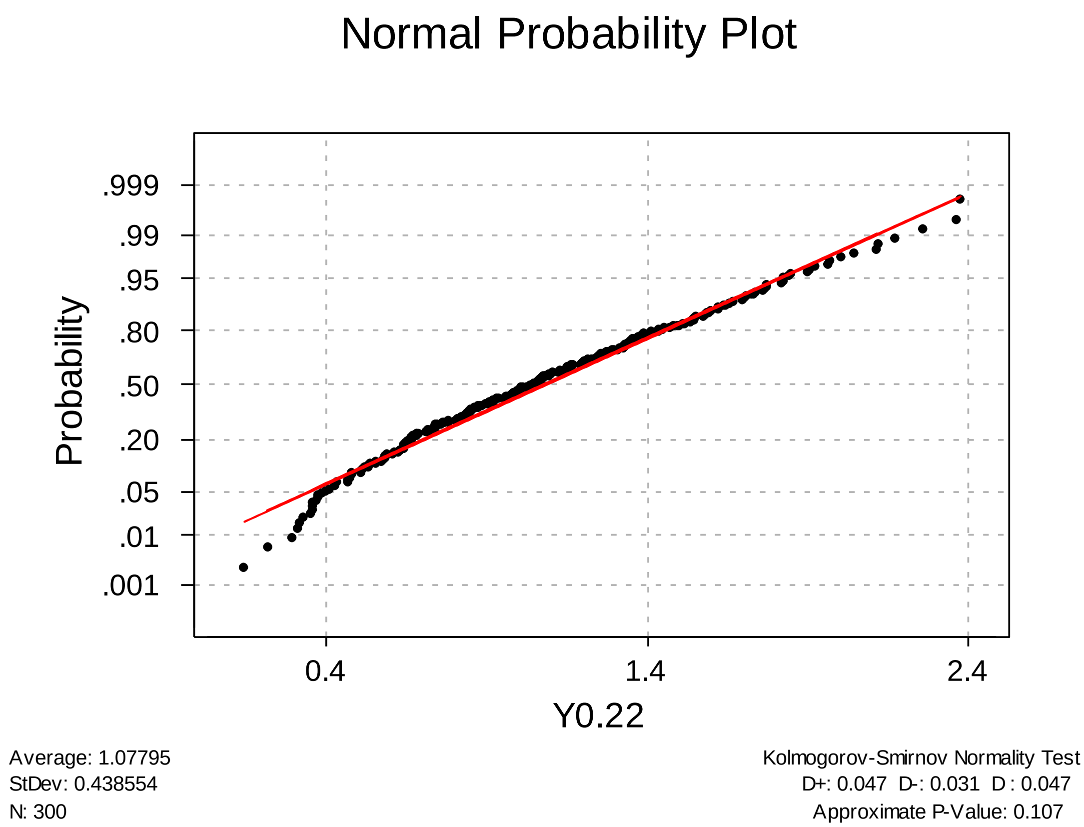
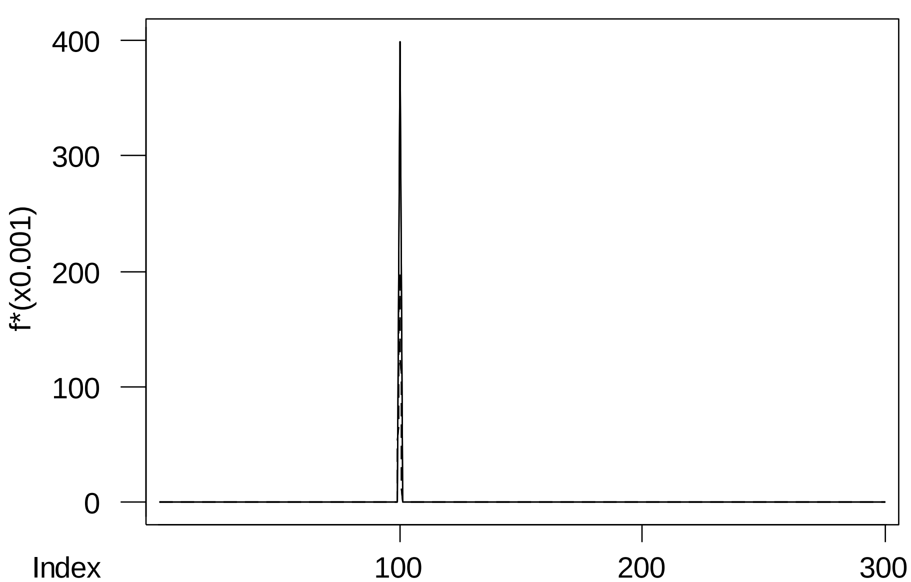
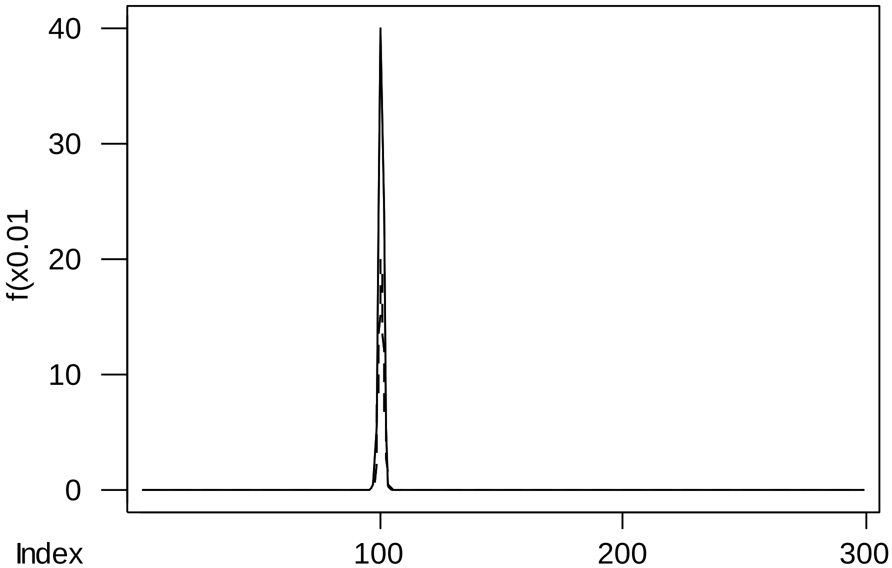
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
0.2
1.2
2.2
Y0.2
Average: 1.07508
StDev: 0.383650
N: 300
Kolmogorov-Smirnov Normality Test
D+: 0.051 D-: 0.030 D : 0.051
Approximate P-Value: 0.061


Normal Probability Plot
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
0.5
1.5
2.5
Y0.21
Average: 1.02281
StDev: 0.425459
N: 300
Kolmogorov-Smirnov Normality Test
D+: 0.050 D-: 0.028 D : 0.050
Approximate P-Value: 0.070


Normal Probability Plot
Probability
.999
.99
.95
.80
.50
.20
.05
.01
.001
0.4
1.4
2.4
Y0.22
Average: 1.07795
StDev: 0.438554
N: 300
Kolmogorov-Smirnov Normality Test
D+: 0.047 D-: 0.031 D : 0.047
Approximate P-Value: 0.107


f*(x0.001)
400
300
200
100
0
Index
100
200
300



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20
10
0
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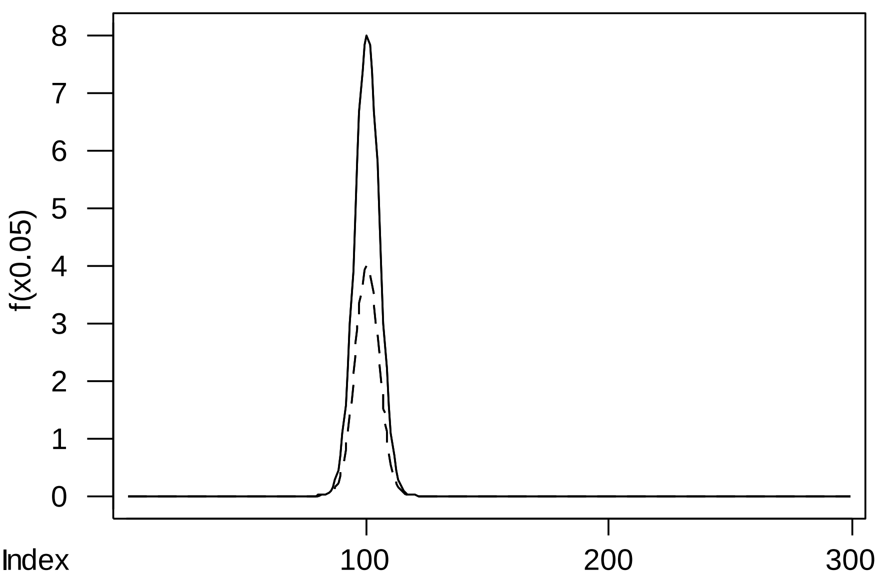

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4
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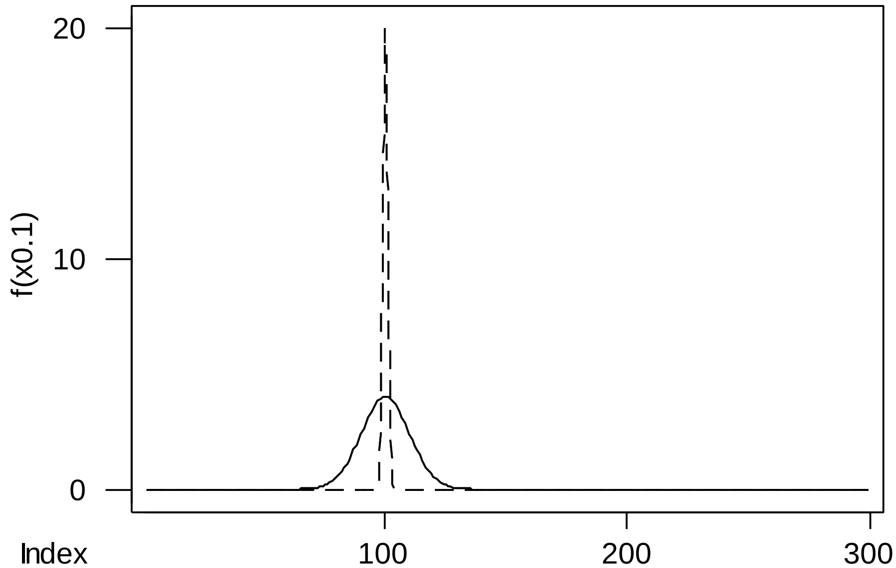
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200
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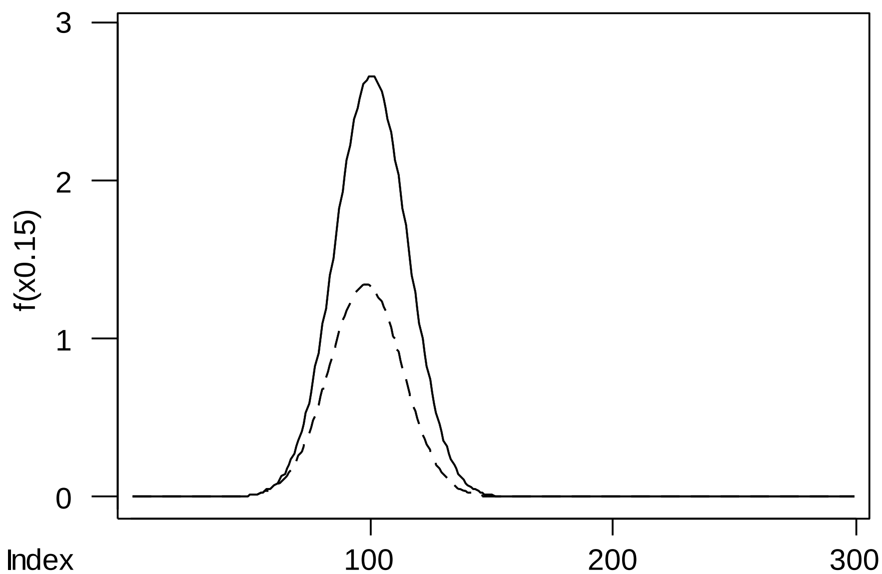
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1
0
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Index
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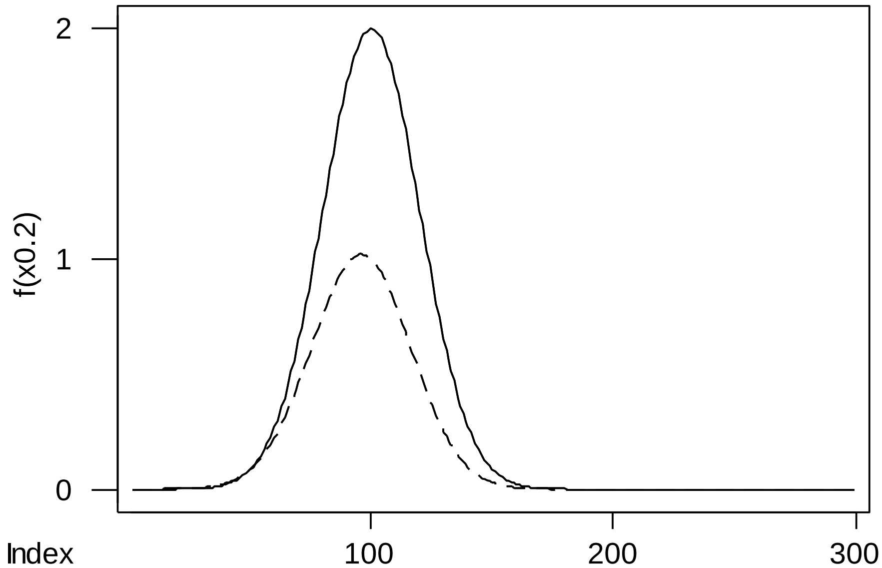
200
300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0
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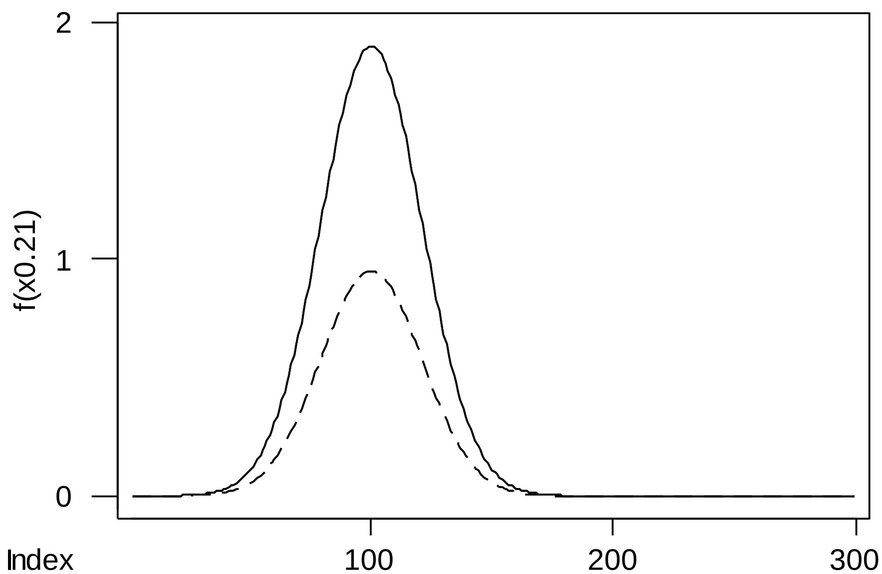
200
300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0
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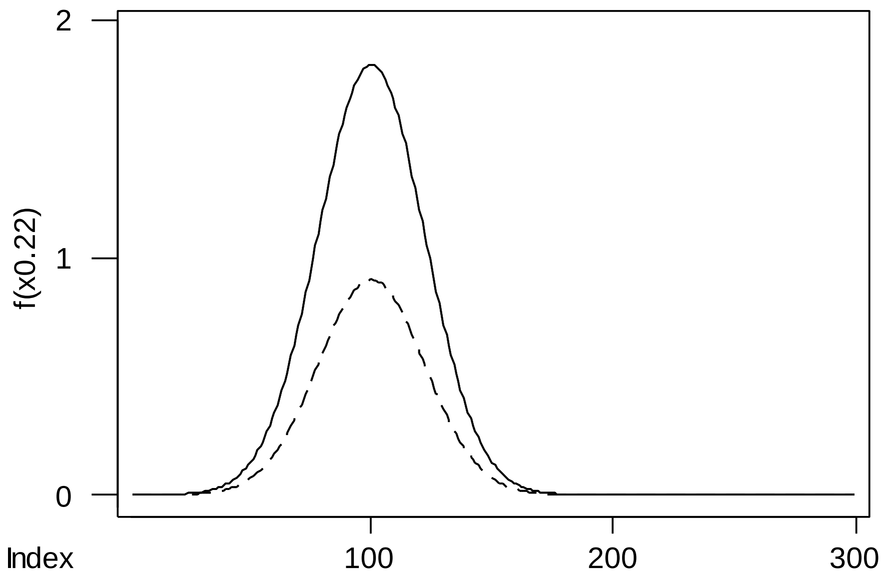
Index
100
200
300


f(x0.22)
2
1
0
Index
100
200
300