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Weibull-Based Process Capability Modeling for Construction Quality Control under Non-Normality: A Reliability–Statistical Framework

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Abstract **Original Research Article**

Conventional process capability indices (Cp, Cpk) assume normality and can yield misleading assessments when construction quality data exhibit skewness and heavy tails. This study develops a Weibull-based capability framework integrating distribution-specific assessment, failure probability, reliability estimation, and bootstrap validation. The methodology was evaluated on 149 concrete compressive strength records with a 20 MPa lower specification limit (LSL). Normality tests confirmed significant departure from normality ($p < 0.001$). Maximum likelihood estimation yielded Weibull shape and scale parameters of 15.6992 and 24.5448. The resulting Weibull Reliability Capability Index (WRCI) was 0.5059, contrasting sharply with the conventional lower capability index (Cpl = 1.0508). The Weibull model revealed a 3.94% nonconformance probability (96.06% reliability). Bootstrap validation (1,000 replications) confirmed metric stability (95% CI: 0.4472–0.6163). The framework provides an accurate, risk-informed quality control alternative without requiring normalizing transformations.

Keywords: concrete compressive strength; process capability; Weibull distribution; non-normality; reliability; failure probability; bootstrap; construction quality control.

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1. Introduction

Process capability analysis is an established statistical approach for evaluating whether a production process can consistently produce outputs within specified engineering limits [8, 17, 27]. Conventional capability indices, particularly Cp and Cpk, have been extensively applied in manufacturing and quality engineering because of their computational simplicity and intuitive interpretation [12, 31, 33]. Their conventional formulation, however, relies substantially on distributional assumptions, especially normality, and on the use of the process mean and standard deviation to characterize process behaviour [2, 17, 31]. The normality assumption becomes problematic when quality characteristics exhibit skewness, kurtosis or asymmetric tails [6, 21, 28, 32]. In such situations, mean–standard-deviation-based capability measures may not adequately

represent the probability of observations occurring near or beyond specification limits [12, 28]. This issue is particularly relevant to construction materials, for which quality characteristics may be influenced by raw-material variability, mix proportions, curing conditions, environmental exposure, and workmanship and production heterogeneity. Concrete compressive strength, for example, may depart from symmetry and exhibit tail behaviour that is not adequately represented by a normal model [30, 31]. Several approaches have been developed to address non-normality in process capability analysis [6, 20, 21, 25, 32]. Transformation procedures provide one important solution by attempting to convert non-normal observations into an approximately normal scale before conventional capability indices are calculated [3, 15, 25, 30, 32]. Although transformation techniques can be useful in normalizing early-age concrete strength data [30, 32], they introduce an additional modeling step, may alter parameter sensitivity, and make the resulting interpretation dependent on the selected transformation function. Distribution-specific approaches provide an alternative by modeling the observed quality characteristic directly according to an appropriate probability distribution [5, 10, 11, 23, 29]. This approach is particularly attractive when the primary concern is accurate representation of asymmetric and tail behaviour [18, 23].

The Weibull distribution is widely established in reliability engineering because of its flexibility in representing different forms of skewness and tail behaviour [9, 13, 14, 16, 24]. Its shape and scale parameters provide a direct probabilistic representation of the underlying distribution [9, 24], while its cumulative distribution function permits direct estimation of failure probabilities and reliability [1, 16, 29]. These characteristics create an opportunity to connect process capability assessment with reliability-oriented quality control [4, 11, 23]. The distinction is particularly important for construction quality assessment: a capability index alone provides a relative measure of process performance against specifications, whereas failure probability directly quantifies the probability of nonconformance [12, 17]. Integrating the two concepts can therefore provide quality-control practitioners with both a capability measure and an explicit estimate of risk.

Despite the extensive use of Weibull models in reliability and engineering applications [1, 9, 18, 24], their integration into construction process capability assessment remains comparatively limited. The present study addresses this methodological gap by developing a Weibull Reliability Capability Index (WRCI) based on distributional percentiles [6, 19, 21] and integrating it with Weibull failure probability, reliability estimation [14, 16] and bootstrap validation [7]. The study therefore contributes a distribution-aware reliability–statistical framework for construction quality control under non-normality. The framework is empirically evaluated using concrete compressive strength data and compared with a conventional lower capability index.

2. Methodology

2.1 Research Design and Data

The study employed a quantitative methodological design combining empirical data analysis, distribution modeling, capability assessment and statistical resampling. The empirical analysis used 149 concrete compressive-strength observations obtained from construction quality-control records. The concrete specification considered in the study was Grade 20, with a lower specification limit (LSL) of 20 MPa**.** The analysis focused on the lower specification because the principal quality-control concern represented in the available data-set was the probability that concrete strength would fall below the minimum required strength.

The methodological sequence was:

1. descriptive assessment of the strength observations;
2. graphical and statistical assessment of normality;
3. Weibull parameter estimation;
4. Weibull percentile estimation;
5. development of the WRCI;

6. estimation of lower-tail failure probability;
7. estimation of reliability;
8. comparison with the conventional Cpl;
9. bootstrap validation of the WRCI.

This sequence forms the proposed hybrid reliability–statistical framework.

2.2 Assessment of Non-Normality

Let X denote concrete compressive strength. Descriptive statistics were used to assess central tendency, variability and distributional shape. Skewness and kurtosis were examined together with graphical diagnostics. Formal normality was evaluated using the Shapiro–Wilk [26] and Anderson–Darling tests [19]. The null hypothesis for both procedures was that the observations were generated from a normal distribution. Evidence of substantial non-normality justified the subsequent use of a distribution-specific model rather than treating the observations as normally distributed [10, 23, 30].

2.3 Weibull Model

The two-parameter Weibull distribution was used to model concrete compressive strength. Its cumulative distribution function is

$$F(x) = 1 - e^{-\left(\frac{x}{\eta}\right)^{\beta}}, \text{ x} \geq 0 \qquad 1$$

Where:

- β > 0 is the shape parameter, which governs the form of the distribution
- η > 0 is the scale parameter, which determines the spread of the distribution .

The corresponding reliability function is

$$R = p(x \geq LSL) = 1 - p_f = \exp\left[-\left(\frac{LSL}{\eta}\right)^{\beta}\right] \qquad 2$$

The Weibull parameters were estimated using maximum likelihood estimation.

2.4 Weibull Reliability Capability Index

Classical capability assessment relies on standard-deviation-based distances from specification limits. Under non-normality, such distances may not adequately characterize asymmetric tail behaviour. The proposed Weibull Reliability Capability Index is defined for a lower specification limit.

Development of a Percentile-Based Weibull Capability Framework

Weibull CDF

$$F(x) = 1 - \exp\left[-\left(\frac{x}{\eta}\right)^{\beta}\right] \qquad 3$$

Quartile function

From Weibull Cumulative Distribution Function (CDF)

Let F(x) = P

Where 0 < p < 1

Substituting p into the CDF

$$P = 1 - \exp\left[-\left(\frac{x}{\eta}\right)^{\beta}\right], x \geq 0 \qquad 4$$

Isolate the exponential term

$$1 - P = \exp\left[-\left(\frac{x}{\eta}\right)^{\beta}\right] \qquad 5$$

Take the natural log on both side

$$In(1 - p) = -\left(\frac{x}{\eta}\right)^{\beta} \qquad 6$$

Multiply both sides by – 1

$$-In(1-p)=\left(\frac{x}{\eta}\right)^{\beta} \quad 7$$

Take the k-th root of both sides

$$[-In(1-p)]^{\frac{1}{\beta}}=\frac{x}{\eta} \quad 8$$

Multiply both sides by η

$$x=\eta[-In(1-p)]^{\frac{1}{\beta}} \quad 9$$

Therefore, the Weibull quartile function is

$$Q(P)=\eta[-In(1-p)]^{\frac{1}{\beta}},\ 0<p<1 \quad 10$$

Where,

Q (p) = P^{-th} Quartile of the Weibull distribution

η = scale parameter

β = shape parameter

P = Cumulative Probability (0 < p < 1)

Using the estimated parameters from the study β = 15.6996, η = 24.5451

The lower tail percentile

$$P_{0.00135} = 24.5451[-In(1-0.00135)]^{\frac{1}{15.6996}} = 16.1138 \quad 11$$

The median percentile

$$P_{0.50} = 24.5451[-In(1-0.50)]^{\frac{1}{15.6996}} = 23.9787 \quad 12$$

Upper percentile

$$P_{0.99865} = 24.5451[-In(1-0.99865)]^{\frac{1}{15.6996}} = 27.6821 \quad 13$$

These Weibull percentiles form the basis for constructing the proposed Weibull Reliability Capability Index (WRCI)

Development of the Weibull Reliability Capability Index (WRCI)

The proposed capability measure was defined as:

$$WRCI = \frac{P_{0..50} - LSL}{P_{0.50} - P_{0.135}} \quad 14$$

Substituting Weibull Quartiles

$$WRCI = \frac{\eta[-In(0.5)]^{\frac{1}{\beta}} - LSL}{\eta[-In(0.5)]^{\frac{1}{\beta}} - \eta[-In(0.99865]^{\frac{1}{\beta}}} \quad 15$$

Substituting the values

Where the standard LSL for 28 days concrete compressive strength was given as 20 by the constructing firm

Hence,

$$WRCI = \frac{23.9987 - 20}{23.9987 - 15.1138} = \frac{3.9987}{7.8847} = 0.5059 \quad 16$$

Reliability Assessment through Failure Probability Modeling

Failure probability

$$P_f = P(\eta < LSL) = F(LSL) = 1 - \exp\left[-\left(\frac{LSL}{\eta}\right)^{\beta}\right] \quad 17$$

Substitute the estimators

$$P_f = 1 - \exp[-\left(\frac{20}{24.5451}\right)^{15.6996}] = 0.03936 \quad 18$$

Reliability function

$$R = p(x \geq LSL) = 1 - p_f = \exp\left[-\left(\frac{LSL}{\eta}\right)^{\beta}\right] \quad 19$$

$$R = 1 - 0.03936 = 0.96064 = 96.06 \quad 20$$

Comparative Evaluation of Classical and Weibull-Based Capability Assessment

Classical Lower Capability Index

$$P_{PL} = \frac{\mu - LSL}{3\sigma} \quad 21$$

Using sample estimates

$$\hat{C}_{pl} = \frac{\bar{X} - LSL}{3S} = \frac{23.909 - 20}{3(1.240)} = 1.0508 \quad 22$$

2.4 Bootstrap Validation

The stability of the proposed WRCI was assessed using non-parametric bootstrap resampling. The procedure involved 1,000 bootstrap replications. For each replication, a sample of 149 observations was drawn with replacement, the Weibull parameters were re-estimated, and the WRCI was recalculated. The bootstrap distribution was summarized using its mean, standard deviation and percentile-based 95% confidence interval.

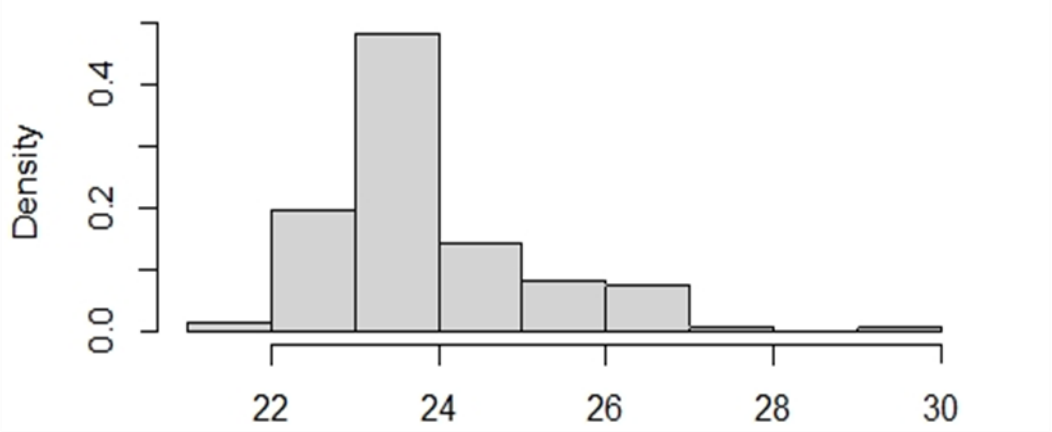
3. Results

3.1 Distributional Characteristics

The empirical data-set contained 149 observations. The mean compressive strength was 23.909 MPa**,** while the median was 23.700 MPa. The standard deviation was 1.240 MPa, with observed strengths ranging from 21.300 to 29.400 MPa.

Table 1. Descriptive statistics of concrete compressive strength

Statistic	Value
Sample size	149
Mean (MPa)	23.909
Median (MPa)	23.700
Standard deviation (MPa)	1.240
Minimum (MPa)	21.300
Maximum (MPa)	29.400
Skewness	1.391
Kurtosis	5.393



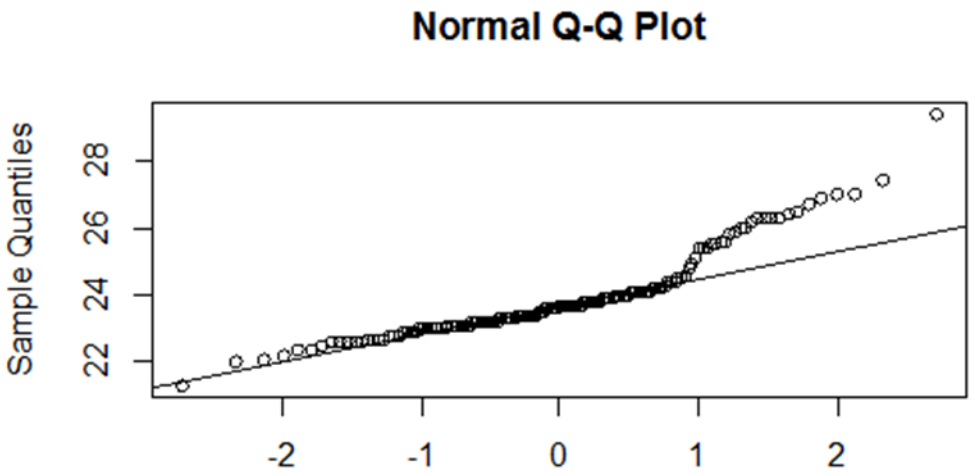
Strength

The table and the graph above shows that positive skewness indicates an asymmetric right tail, while the kurtosis value above the normal benchmark indicates heavier-tailed behaviour. The difference between the mean and median provides additional evidence of asymmetry. The formal normality assessment in the project also rejected normality. Consequently, the empirical distribution does not adequately satisfy the assumptions underlying conventional normal-theory capability assessment.

Statistical Assessment of Normality

Table 2. Normality Test Results

Test	Statistic	p-value
Shapiro–Wilk	0.8808	1.362×10^{-9}
Anderson–Darling	6.2777	1.733×10^{-15}



Discussion

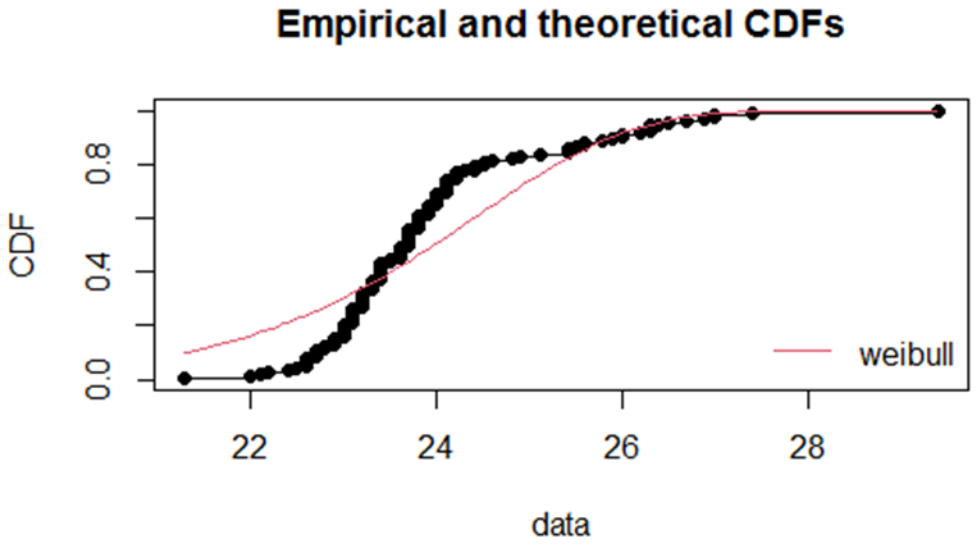
Both tests strongly reject the null hypothesis of normality at the 5% significance level. The extremely small p-values provide compelling statistical evidence that the concrete strength data are non-normal.

3.2 Weibull Parameter Estimation

Maximum likelihood estimation produced the following Weibull parameters:

Table 3. Estimated Weibull parameters

Parameter	Estimate
Shape, β(beta)	15.6992
Scale, η(eta)	24.5448



The estimated shape parameter indicates the form of the empirical Weibull distribution, while the scale parameter provides its characteristic strength level. The Weibull model was subsequently used to obtain the distributional percentiles required for capability and reliability assessment.

3.3 Weibull Percentiles

Table 4. Selected Weibull percentiles

Percentile	Strength (MPa)
$Q_{0.00135}$	16.1138
$Q_{0.5}$	23.9787
$Q_{0.99985}$	27.6821

The median of the fitted distribution was approximately 23.98 MPa, exceeding the specified minimum strength of 20 MPa. However, the lower-tail percentile is substantially below the specification limit. This difference illustrates why the central tendency alone is insufficient for

assessing construction quality risk. The lower-tail characteristics of the fitted distribution are particularly important because the quality specification is one-sided.

3.4 Weibull Reliability Capability Index

Using the lower specification limit of 20 MPa, the proposed WRCI was estimated as:

WRCI=0.5059

Table 5. Capability comparison

Capability measure	Value
Classical Cpl	1.0508
Weibull WRCI	0.5059

The two measures provide substantially different numerical representations of process performance. The conventional Cpl is greater than one when calculated under the normal-theory formulation. However, the earlier distributional analysis established substantial non-normality. Thus, its interpretation should be made cautiously because the assumptions underlying the conventional benchmark are not supported by the observed data. The WRCI, in contrast, is constructed directly from Weibull percentiles and therefore incorporates the fitted distribution's asymmetry and tail structure.

3.5 Failure Probability and Reliability

The fitted Weibull model produced a lower-tail failure probability of

$Pf = 0.03936$

Thus,

$Pf = 3.94\%$ and the corresponding reliability is $R = 1-Pf = 0.96064$ or

R=96.06%.

Table 6. Weibull reliability assessment

Measure	Estimate
Lower specification limit	20 MPa
Failure probability	0.03936
Failure probability (%)	3.94
Reliability	0.96064
Reliability (%)	96.06

The results indicate that approximately 3.94% of observations are expected to fall below the 20 MPa specification under the fitted Weibull model, while approximately 96.06% are expected to meet or exceed the specification. This result adds information that is not directly expressed by the conventional capability index.

3.6 Bootstrap Validation

The bootstrap analysis was based on 1,000 resamples.

Table 7. Bootstrap validation of WRCI

Measure	Estimate
Bootstrap mean	0.5219
Bootstrap standard deviation	0.0468
95% CI lower limit	0.4472
95% CI upper limit	0.6163

The bootstrap estimates remained within a relatively defined interval around the original WRCI estimate. The resulting confidence interval, (0.4472,0.6163) provides empirical information on the sampling variability of the proposed capability measure. The bootstrap analysis therefore supports the statistical stability of the WRCI within the empirical setting examined in this study. However, the interval should not be interpreted as proof of universal robustness across construction data-set, since the framework was validated using one empirical dataset.

4. Discussion

The findings provide empirical support for the use of distribution-specific capability modelling when construction quality data depart materially from normality. The concrete-strength observations exhibited positive skewness and excess kurtosis, while formal normality assessment rejected the normal distribution. These characteristics are important because conventional capability indices summarize process behaviour through the mean and standard deviation and are commonly interpreted using normal-theory probability relationships which the results may be misleading. The estimated Weibull model provided a direct representation of the observed strength distribution. Rather than transforming the strength measurements to approximate normality, the proposed approach fitted the Weibull distribution to the observed measurements and derived capability information from its percentiles. This preserves the original engineering scale of the concrete-strength measurements. The comparison between Cpl=1.0508C and WRCI =0.5059 is particularly informative. The conventional index exceeds one, which under its normal-theory interpretation indicates that the lower specification is relatively distant from the process mean compared with three standard deviations. However, this interpretation does not explicitly quantify the actual lower-tail probability under the observed non-normal distribution. The Weibull analysis estimated a 3.94% probability of strength falling below the 20 MPa specification. This finding demonstrates the value of combining capability measurement with explicit reliability analysis. Capability indices and failure probabilities answer related but different questions: the former provides a standardized measure of process performance relative to specifications, while the latter directly quantifies the probability of non-conformance.

The WRCI further incorporates the lower-tail percentile into the capability calculation. This is important in one-sided construction specifications because the engineering concern is not merely how far the mean lies from the specification but how the distribution behaves near the failure boundary. The bootstrap analysis strengthens the empirical evaluation by examining the sampling stability of the proposed index. The bootstrap mean of 0.5219 was reasonably close to the original WRCI of 0.5059, while the 95% confidence interval extended

from 0.4472 to 0.6163. This indicates that the estimated capability measure is subject to sampling variability but remains estimable within a defined empirical range which further shows that for a non-normal data-set classical cpl is misleading and this proposed framework can serve as an alternative method in solving non-normal data-set especially for a construction firm. Taken together, the results support the central proposition of the framework: construction quality capability assessment under non-normality can be strengthened by integrating distribution-specific modeling, percentile-based capability assessment, failure probability and reliability estimation, and bootstrap uncertainty assessment. Importantly, the contribution of the study is not the invention of the Weibull distribution. Rather, the contribution lies in adapting Weibull modeling into an integrated construction-quality capability framework in which capability and reliability are evaluated jointly.

5. Proposed Hybrid Reliability–Statistical Framework

Based on the empirical findings, the proposed framework can be represented as a sequential quality-control procedure:

Construction quality data

↓

Distributional screening

Descriptive statistics → graphical assessment → formal normality tests

↓

Decision on distributional suitability

Normality supported → conventional capability methods may be considered

Normality rejected → distribution-specific modeling

↓

Weibull parameter estimation

Maximum likelihood estimation of β and η

↓

Distributional percentiles

$Q_{0.00135}, Q_{0.50}, Q_{0.99865}$

↓

Weibull capability assessment

WRCI

↓

Reliability assessment

Failure probability + reliability

↓

Benchmark comparison

Classical Cpl versus Weibull-based capability measure

↓

Uncertainty assessment

1,000 bootstrap replications + confidence interval

↓

Risk-informed construction quality decision

This workflow is the principal methodological contribution of the study. It connects statistical distribution assessment with engineering reliability rather than treating capability analysis as an isolated numerical calculation.

6. Conclusion

This study developed and empirically evaluated a Weibull-based process capability framework for construction quality control under non-normality. Using 149 concrete compressive-strength observations, the analysis established substantial departures from normality and demonstrated the limitations of relying exclusively on conventional normal-based capability assessment. Maximum likelihood estimation produced Weibull shape and scale parameters of 15.6992 and 24.5448, respectively. The resulting Weibull percentiles were used to construct a Weibull Reliability Capability Index of 0.5059. For comparison, the conventional lower capability index was 1.0508. The fitted Weibull model additionally estimated a lower-tail failure probability of 3.94% at the 20 MPa specification limit, corresponding to 96.06% reliability. These results demonstrate the practical

value of explicitly quantifying non-conformance risk alongside capability. Bootstrap validation based on 1,000 replications produced a mean WRCI of 0.5219, standard deviation of 0.0468 and a 95% confidence interval of 0.4472–0.6163. This provides empirical evidence of the sampling behaviour and stability of the proposed index within the study data-set. The principal contribution of the study is therefore a hybrid reliability–statistical framework that integrates non-normal distribution modeling, percentile-based capability assessment, failure probability, reliability and bootstrap validation. The framework provides a direct alternative to approaches that first transform construction quality measurements toward normality. Nevertheless, the findings should be interpreted within the study's empirical scope. The framework was evaluated using one concrete-strength data-set and a lower specification limit. Broader validation using multiple construction materials, projects, curing ages and two-sided specification settings is required before generalization.

7. Practical Implications

The proposed framework has several implications for construction quality management.

First, **distributional assessment should precede capability analysis**. Applying normal-based capability indices without examining distributional characteristics can obscure important tail behaviour.

Second, **failure probability should accompany capability indices** where quality failures have engineering consequences. A capability index alone does not directly communicate the probability of non-conformance.

Third, **distribution-specific capability measures can preserve the original measurement scale.** This is useful for engineering communication because the strength measurements remain expressed in MPa rather than being interpreted only through a transformed scale.

Finally, **bootstrap validation can provide useful uncertainty information** for capability measures whose analytical sampling distributions may be difficult to obtain.

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